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Solving Neutrosophic Travelling Salesman Problem using Pentagonal Fuzzy Number

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Abstract: Travelling salesman problem is well - known studied problem. In this paper we discuss a Neutrosophic pentagonal fuzzy numbers which are more realistic in nature. Neutrosophic pentagonal fuzzy numbers can converted to a Neutrosophic fuzzy numbers then the problem is solved by ones assignment method. Neutrosophic crisp traveling salesman problem also solved by using a score function and then find the optimum solution. Numerical examples are included for a way.

Keywords: Fuzzy set, Neutrosophic set, Neutrosophic Pentagonal fuzzy set, Fuzzy travelling salesman algorithm, score function, Ranking function.

INTRODUCTION

The travelling salesman problem is the well - known problem in the field of computational mathematics. A salesman who wants to travel a city only once in a map and return to his home town. In such a way that the length of the journey is two short among all other possible journey. The Neutrosophic set was first proposed by F. Smarandache in 1995. The evolution of Neutrosophic components are truth, indeterminacy and falsity membership values respectively and it is non-standard unit interval. Here, we generate two methods to solve the Neutrosophic travelling salesman problem. The Neutrosophic Pentagonal fuzzy numbers can be converted to a Neutrosophic fuzzy numberusing a ranking function. Neutrosophic crisp travelling salesman problem also solved by using score function.

PRELIMINARIES

Definition 1: Fuzzy set

A fuzzy set is characterized by its membership function mapping the element of a domain, space or universe of discourse mapped into the unit interval [0,1]. A fuzzy set A in the universal set X is defined as A = {(x, μ_A(x)) / x∈ X}. Here, $\mu_A(x) : X \to [0,1]$ is a mapping called the degree of membership function and μ_A(x) is called the membership value of x∈X in the fuzzy set A. These membership grades are often represented by real numbers ranging [0,1].

Definition 2: Normal fuzzy set

A fuzzy set A is called normal if there exists an element x∈X whose membership value is one, i.eμ_A(x) = 1.

Definition 3: Fuzzy number

A fuzzy set A of real line R with membership function $\mu_A(x) : R \to [0,1]$ is called fuzzy number if

(i) A is normal and convexity.
(ii) A must be bounded.
(iii) μ_A(x) is piecewise continuous.

Definition 4:Membership function

Let X denote the universal set of the membership function

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Definition 5: Triangular fuzzy number

A fuzzy number A = (a_1, a_2, a_3) is said to be triangular fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \end{cases}$$

0, otherwise

Where $a_1 \leq a_2 \leq a_3$ are real numbers.

Definition 6:Trapezoidal fuzzy number

A fuzzy number A = (a_1, a_2, a_3, a_4) is called a trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 < x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_3-x}{a_4-a_3}, & a_3 < x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

Where $a_1 \leq a_2 \leq a_3 \leq a_4$ are real numbers.

Definition 7:Pentagonal fuzzy number

A fuzzy number A = (a_1, a_2, a_3, a_4, a_5) is said to be a pentagonal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 1, & x = a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ \frac{a_5-x}{a_5-a_4}, & a_4 \leq x \leq a_5 \\ 0, & x > a_5 \end{cases}$$

Where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ are real numbers.

Definition 8:Ranking function

A ranking function $\mathcal{R}$: F (R) → R which maps each fuzzy numbers to real line F (R) represent the set of all pentagonal fuzzy number. If R be any linear ranking function

$$\mathcal{R}_{aij}^{T} = \frac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$$

Definition 9:Neutrosophic set

Let X be a universe. A Neutrosophic set A over X is defined by

A^N = { (x: $T_{a^N}(\mathbf{x})$, $I_{a^N}(\mathbf{x})$,F_{a^N}(x)) : x∈ X} where T_{a^N} , I_{a^N} ,F_{a^N} : X → $]0^-, 3^+[$ are called the truth, indeterminacy and falsity membership function of the element x∈ X to the setA^N with $0^- \leq T_{a^N}(\mathbf{x}) + I_{a^N}(\mathbf{x}) + F_{a^N}(x) \leq 3^+$.

Definition 10:NeutrosophicPentagonal fuzzy set

Let X be the finite universe of discourse and $F^N[0,1]$ denoted by the set of all pentagonal fuzzy numbers on [0,1]. A Neutrosophic pentagonal fuzzy set A in X is represented by A^N = { (x: $T_{a^N}(\mathbf{x})$, $I_{a^N}(\mathbf{x})$,F_{a^N}(x)) : x∈ X}.

Where the Neutrosophicpentagonal fuzzy numbers,

$T_{a^N}(\mathbf{x}) = (\boldsymbol{T}_{a_1}^N(\mathbf{x}), \boldsymbol{T}_{a_2}^N(\mathbf{x}), \boldsymbol{T}_{a_3}^N(\mathbf{x}), \boldsymbol{T}_{a_4}^N(\mathbf{x}), \boldsymbol{T}_{a_5}^N(\mathbf{x}))$,

$I_{a^N}(\mathbf{x}) = (\boldsymbol{I}_{a_1}^N(\mathbf{x}), \boldsymbol{I}_{a_2}^N(\mathbf{x}), \boldsymbol{I}_{a_3}^N(\mathbf{x}), \boldsymbol{I}_{a_4}^N(\mathbf{x}), \boldsymbol{I}_{a_5}^N(\mathbf{x}))$,

$F_{a^N}(\mathbf{x}) = (\boldsymbol{F}_{a_1}^N(\mathbf{x}), \boldsymbol{F}_{a_2}^N(\mathbf{x}), \boldsymbol{F}_{a_3}^N(\mathbf{x}), \boldsymbol{F}_{a_4}^N(\mathbf{x}), \boldsymbol{F}_{a_5}^N(\mathbf{x}))$ be the truth , indeterminacy and falsity membership degree of x in A and for every x∈ Xsuch that

$0^- \leq T_{a^N}(\mathbf{x}) + I_{a^N}(\mathbf{x}) + F_{a^N}(x) \leq 3^+$.

Ranking of Neutrosophic pentagonal fuzzy number is given by

$$\mathcal{R}_{aij}^{T} = T_{a^N}(\mathbf{x}) = \frac{(\boldsymbol{T}_{a_1}^N(\mathbf{x}) + \boldsymbol{T}_{a_2}^N(\mathbf{x}) + \boldsymbol{T}_{a_3}^N(\mathbf{x}) + \boldsymbol{T}_{a_4}^N(\mathbf{x}) + \boldsymbol{T}_{a_5}^N(\mathbf{x}))}{5}$$

$$\mathcal{R}_{aij}^{T} = I_{a^N}(\mathbf{x}) = \frac{(\boldsymbol{I}_{a_1}^N(\mathbf{x}) + \boldsymbol{I}_{a_2}^N(\mathbf{x}) + \boldsymbol{I}_{a_3}^N(\mathbf{x}) + \boldsymbol{I}_{a_4}^N(\mathbf{x}) + \boldsymbol{I}_{a_5}^N(\mathbf{x}))}{5}$$

$$\mathcal{R}_{aij}^{T} = F_{a^N}(\mathbf{x}) = \frac{(\boldsymbol{F}_{a_1}^N(\mathbf{x}) + \boldsymbol{F}_{a_2}^N(\mathbf{x}) + \boldsymbol{F}_{a_3}^N(\mathbf{x}) + \boldsymbol{F}_{a_4}^N(\mathbf{x}) + \boldsymbol{F}_{a_5}^N(\mathbf{x}))}{5}$$

Neutrosophic Fuzzy Travelling Salesman Problem

Suppose a salesman has to visits n cities. He visits one particular city and return to the home town within a short period of time. The Neutrosophicfuzzy travelling salesman problem in the following matrix formulated as

City → ↓	1	2	⋯	J	⋯	N
1	∞	$T_{12}^N, I_{12}^N, F_{12}^N$	⋯	$T_{1j}^N, I_{1j}^N, F_{1j}^N$	⋯	$T_{1n}^N, I_{1n}^N, F_{1n}^N$
2	$T_{21}^N, I_{21}^N, F_{21}^N$	∞	⋯	$T_{2j}^N, I_{2j}^N, F_{2j}^N$	⋯	$T_{2n}^N, I_{2n}^N, F_{2n}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
J	$T_{j1}^N, I_{j1}^N, F_{j1}^N$	$T_{j2}^N, I_{j2}^N, F_{j2}^N$	⋯	∞	⋯	$T_{jn}^N, I_{jn}^N, F_{jn}^N$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
N	$T_{n1}^N, I_{n1}^N, F_{n1}^N$	$T_{n2}^N, I_{n2}^N, F_{n2}^N$	⋯	$T_{nj}^N, I_{nj}^N, F_{nj}^N$	⋯	∞

The Neutrosophic fuzzy travelling salesman problem algorithm

The Neutrosophic fuzzy travelling salesman problem is given by,

$$\begin{pmatrix} (T_{a_{11}}^N, I_{a_{11}}^N, F_{a_{11}}^N) & (T_{a_{12}}^N, I_{a_{12}}^N, F_{a_{12}}^N) & (T_{a_{13}}^N, I_{a_{13}}^N, F_{a_{13}}^N) & \cdots & (T_{a_{1n}}^N, I_{a_{1n}}^N, F_{a_{1n}}^N) \\ (T_{a_{21}}^N, I_{a_{21}}^N, F_{a_{21}}^N) & (T_{a_{22}}^N, I_{a_{22}}^N, F_{a_{22}}^N) & (T_{a_{23}}^N, I_{a_{23}}^N, F_{a_{23}}^N) & \cdots & (T_{a_{2n}}^N, I_{a_{2n}}^N, F_{a_{2n}}^N) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (T_{a_{n1}}^N, I_{a_{n1}}^N, F_{a_{n1}}^N) & (T_{a_{n2}}^N, I_{a_{n2}}^N, F_{a_{n2}}^N) & (T_{a_{n3}}^N, I_{a_{n3}}^N, F_{a_{n3}}^N) & \cdots & (T_{a_{nn}}^N, I_{a_{nn}}^N, F_{a_{nn}}^N) \end{pmatrix}$$

Step1: The Neutrosophic pentagonal fuzzy element can be converted to a Neutrosophic fuzzy numbers by using ranking of Neutrosophic pentagonal fuzzy number.

Step 2: In a Minimization case, Findthe minimum element of each row in the distance matrix(say$a$) and write it on the right side.

$$\begin{pmatrix} (T_{a_{11}}^N, I_{a_{11}}^N, F_{a_{11}}^N) & (T_{a_{12}}^N, I_{a_{12}}^N, F_{a_{12}}^N) & (T_{a_{13}}^N, I_{a_{13}}^N, F_{a_{13}}^N) & \cdots & (T_{a_{1n}}^N, I_{a_{1n}}^N, F_{a_{1n}}^N) \\ (T_{a_{21}}^N, I_{a_{21}}^N, F_{a_{21}}^N) & (T_{a_{22}}^N, I_{a_{22}}^N, F_{a_{22}}^N) & (T_{a_{23}}^N, I_{a_{23}}^N, F_{a_{23}}^N) & \cdots & (T_{a_{2n}}^N, I_{a_{2n}}^N, F_{a_{2n}}^N) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (T_{a_{n1}}^N, I_{a_{n1}}^N, F_{a_{n1}}^N) & (T_{a_{n2}}^N, I_{a_{n2}}^N, F_{a_{n2}}^N) & (T_{a_{n3}}^N, I_{a_{n3}}^N, F_{a_{n3}}^N) & \cdots & (T_{a_{nn}}^N, I_{a_{nn}}^N, F_{a_{nn}}^N) \end{pmatrix} \begin{matrix} a_1^N \\ a_2^N \\ \vdots \\ a_n^N \end{matrix}$$

Then divide each element of i^{th} row of the matrix by a . This will create atleast one ones in each row.

$$\begin{pmatrix} (T_{a_{11}}^N, I_{a_{11}}^N, F_{a_{11}}^N)/a_1^N & (T_{a_{12}}^N, I_{a_{12}}^N, F_{a_{12}}^N)/a_1^N & (T_{a_{13}}^N, I_{a_{13}}^N, F_{a_{13}}^N)/a_1^N & \cdots & (T_{a_{1n}}^N, I_{a_{1n}}^N, F_{a_{1n}}^N)/a_1^N \\ (T_{a_{21}}^N, I_{a_{21}}^N, F_{a_{21}}^N)/a_2^N & (T_{a_{22}}^N, I_{a_{22}}^N, F_{a_{22}}^N)/a_2^N & (T_{a_{23}}^N, I_{a_{23}}^N, F_{a_{23}}^N)/a_2^N & \cdots & (T_{a_{2n}}^N, I_{a_{2n}}^N, F_{a_{2n}}^N)/a_2^N \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (T_{a_{n1}}^N, I_{a_{n1}}^N, F_{a_{n1}}^N)/a_n^N & (T_{a_{n2}}^N, I_{a_{n2}}^N, F_{a_{n2}}^N)/a_n^N & (T_{a_{n3}}^N, I_{a_{n3}}^N, F_{a_{n3}}^N)/a_n^N & \cdots & (T_{a_{nn}}^N, I_{a_{nn}}^N, F_{a_{nn}}^N)/a_n^N \end{pmatrix} \begin{matrix} a_1^N \\ a_2^N \\ \vdots \\ a_n^N \end{matrix}$$

Step 3: Find the Minimum element of each column in the distance matrix(say b_j^N) and write it belowj^{th} column. Then divide each element ofj^{th} column of the matrix by b . This will create atleast a one ones in each column. Then make assignments in terms of ones.

$$\begin{pmatrix} (T_{a_{11}}^N, I_{a_{11}}^N, F_{a_{11}}^N)/a_1^N b_1^N & (T_{a_{12}}^N, I_{a_{12}}^N, F_{a_{12}}^N)/a_1^N b_2^N & (T_{a_{13}}^N, I_{a_{13}}^N, F_{a_{13}}^N)/a_1^N b_3^N & \cdots & (T_{a_{1n}}^N, I_{a_{1n}}^N, F_{a_{1n}}^N)/a_1^N b_n^N \\ (T_{a_{21}}^N, I_{a_{21}}^N, F_{a_{21}}^N)/a_2^N b_1^N & (T_{a_{22}}^N, I_{a_{22}}^N, F_{a_{22}}^N)/a_2^N b_2^N & (T_{a_{23}}^N, I_{a_{23}}^N, F_{a_{23}}^N)/a_2^N b_3^N & \cdots & (T_{a_{2n}}^N, I_{a_{2n}}^N, F_{a_{2n}}^N)/a_2^N b_n^N \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ (T_{a_{n1}}^N, I_{a_{n1}}^N, F_{a_{n1}}^N)/a_n^N b_1^N & (T_{a_{n2}}^N, I_{a_{n2}}^N, F_{a_{n2}}^N)/a_n^N b_2^N & (T_{a_{n3}}^N, I_{a_{n3}}^N, F_{a_{n3}}^N)/a_n^N b_3^N & \cdots & (T_{a_{nn}}^N, I_{a_{nn}}^N, F_{a_{nn}}^N)/a_n^N b_n^N \end{pmatrix} \begin{matrix} a_1^N \\ a_2^N \\ \vdots \\ a_n^N \end{matrix}$$

$$b_1^N b_2^N b_3^N \cdots b_n^N$$

Step 4: Obtain the optimum schedule and minimum cost for theNeutrosophic fuzzy travelling salesman problem.

THE NEUTROSOPHIC CRISP TRAVELLING SALESMAN PROBLEM ALGORITHM

Step 1: Construct the cost matrix ofNeutrosophic fuzzy travelling salesman problem, $A^N = (C_{ij})_{\times n}$

Step 2: Determine the Evaluation Neutrosophic matrix as $E^N(A) = [() , (T^N)^u]$

$$\text{Where, } [(T^N)^l , (T^N)^u] = [min((\frac{T^N+I^N}{2}),(\frac{1-F^N+I^N}{2})),max((\frac{T^N+I^N}{2}),(\frac{1-F^N+I^N}{2}))]$$

Step 3: Compute the score function $S^N(A)$ of an alternative $S^N(A) = 2[()^u , (T^N)^l]$

i.e., $S^N(A) = 2\,[max((\frac{T^N+I^N}{2}),(\frac{1-F^N+I^N}{2}))\text{-}min((\frac{T^N+I^N}{2}),(\frac{1-F^N+I^N}{2}))]$
where $0 \le S^N(A) \le 1$.

Step 4: Take the score function matrix as initial input data for Neutrosophic crisp travelling salesman problem and solve it by ones assignment method.

NUMERICAL EXAMPLE
Method: 1-Solving Neutrosophic fuzzy travelling salesman problem

Let us consider the following travelling salesman problem. A company has four cities. The working salesman has to distribute the products from one city to other. The cost of the travel from cities A,B,C,D to A,B,C,D is given below.

	A	B	C	D
A	∞	[(1,2,3,4,5);(2,3,4,5,6);(3,4,5,6,7)]	[(0,1,2,3,4);(1,2,3,4,5);(2,3,4,5,6)]	[(2,3,4,5,6);(3,4,5,6,7);(4,5,6,7,8)]
B	[(0,1,2,3,4);(0,2,4,6,8);(2,4,6,8,10)]	∞	[(1,2,5,7,10);(2,5,7,9,12);(5,7,9,11,13)]	[(2,3,5,7,8);(3,5,6,7,9);(5,6,7,8,9)]
C	[(0,2,4,6,8);(2,4,6,8,10);(4,6,8,10,12)]	[(3,4,5,6,7);(4,5,6,7,8);(5,6,7,8,9)]	∞	[(0,1,2,3,4);(2,3,4,5,6);(3,4,5,6,7)]
D	[(0,1,3,5,6);(1,2,4,6,7);(2,4,6,8,10)]	[(1,2,3,4,5);(2,3,4,5,11);(4,5,6,7,8)]	[(0,2,4,6,8);(0,3,6,9,12);(3,6,9,10,12)]	∞

Solution

Step 1: To convert the given probleminto Neutrosophicfuzzy number by using ranking pentagonal fuzzy number

$R^T(a_{ij})$	TRUTH MEMBERHIP	INDETERMINACY	FALSITY
$R^T(a_{12})$	3	4	5
$R^T(a_{13})$	2	3	4
$R^T(a_{14})$	4	5	6
$R^T(a_{21})$	2	4	6
$R^T(a_{23})$	5	7	9
$R^T(a_{24})$	5	6	7
$R^T(a_{31})$	4	6	8
$R^T(a_{32})$	5	6	7
$R^T(a_{34})$	2	4	5
$R^T(a_{41})$	3	4	6
$R^T(a_{42})$	3	5	6
$R^T(a_{43})$	4	6	8

Ranking of Neutrosophic pentagonal fuzzy number

$$\mathcal{R}^T_{aij} = T_{a^N}(\mathbf{x}) = \frac{(T^N_{a_1}(\mathbf{x}) + T^N_{a_2}(\mathbf{x}) + T^N_{a_3}(\mathbf{x}) + T^N_{a_4}(\mathbf{x}) + T^N_{a_5}(\mathbf{x}))}{5}$$

$$\mathcal{R}^T_{aij} = I_{a^N}(\mathbf{x}) = \frac{(I^N_{a_1}(\mathbf{x}) + I^N_{a_2}(\mathbf{x}) + I^N_{a_3}(\mathbf{x}) + I^N_{a_4}(\mathbf{x}) + I^N_{a_5}(\mathbf{x}))}{5}$$

$$\mathcal{R}^T_{aij} = F_{a^N}(\mathbf{x}) = \frac{(F^N_{a_1}(\mathbf{x}) + F^N_{a_2}(\mathbf{x}) + F^N_{a_3}(\mathbf{x}) + F^N_{a_4}(\mathbf{x}) + F^N_{a_5}(\mathbf{x}))}{5}$$

Truth = $R^T(a_{12}) = \frac{1+2+3+4+5}{5} = \frac{15}{5} = \mathbf{3}$ **;** Indeterminacy = $R^T(a_{12}) = \frac{2+3+4+5+6}{5} = \frac{20}{5} = \mathbf{4}$

Falsity = $R^T(a_{12}) = \frac{3+4+5+6+7}{5} = \frac{25}{5} = \mathbf{5}$

Step 2: The Neutrosophic fuzzy travelling salesman problem is given by,

	A	B	C	D
A	∞	(3,4,5)	(2,3,4)	(4,5,6)
B	(2,4,6)	∞	(5,7,9)	(5,6,7)
C	(4,6,8)	(5,6,7)	∞	(2,4,5)

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D	(3,4,6)	(3,5,6)	(4,6,8)	∞

Step 3: Find out the minimum Neutrosophic element of each row in the distance matrix(saya_i^N) and put it on the right side of the matrix,

	A	B	C	D	
A	∞	(3,4,5)	(2,3,4)	(4,5,6)	(2,3,4)
B	(2,4,6)	∞	(5,7,9)	(5,6,7)	(2,4,6)
C	(4,6,8)	(5,6,7)	∞	(2,4,5)	(2,4,5)
D	(3,4,6)	(3,5,6)	(4,6,8)	∞	(3,4,6)

Then divide each element of i^{th} row of the matrix by a . Each row creates some ones and the matrix will reduce to the following.

	A	B	C	D	
A	∞	(1.5,1.33,1.25)	1^N	(2,1.66,1.5)	(2,3,4)
B	1^N	∞	(2.5,1.75,1.5)	(2.5,1.5,1.16)	(2,4,6)
C	(2,1.5,1.6)	(2.5,1.5,1.4)	∞	1^N	(2,4,5)
D	1^N	(1,1.25,1)	(1.33,1.5,1.33)	∞	(3,4,6)

Step 4: Now find the minimum element of each column in distance matrix (say b_j^N) and divide each element of j^{th} column of the matrix by b . This will create some ones to each row and column.

	A	B	C	D	
A	∞	(1.5,1.064,1.25)	1^N	(2,1.66,1.5)	(2,3,4)
B	1^N	∞	(2.5,1.75,1.5)	(2.5,1.5,1.16)	(2,4,6)
C	(2,1.5,1.6)	(2.5,1.2,1.4)	∞	1^N	(2,4,5)
D	1^N	1^N (1,1.25,1)	(1.33,1.5,1.33)	∞	(3,4,6)

Step 5: The optimum schedule is given by

	A	B	C	D	
A	∞	(1.5,1.064,1.25)	[1^N]	(2,1.66,1.5)	(2,3,4)
B	[1^N]	∞	(2.5,1.75,1.5)	(2.5,1.5,1.16)	(2,4,6)
C	(2,1.5,1.6)	(2.5,1.2,1.4)	∞	[1^N]	(2,4,5)
D	1^N	[1^N] (1,1.25,1)	(1.33,1.5,1.33)	∞	(3,4,6)

Step 6: City A assigns to City C Distance (2,3,4)
City B assigns to City A Distance (2,4,6)
City C assigns to City D Distance (2,4,5)
City D assigns to City B Distance (3,5,6)

Therefore, the root conditions aresatisfied and is given by

A →C , B→A , C → D , D → B

A → C → D → B → A

The Neutrosophic fuzzy optimum solution is given by

(2,3,4) + (2,4,5) + (3,5,6) + (2,4,6) = (9,16,21)

Method 2:Solving Neutrosophic crisp travelling salesman problem

Step 1: Inthe above example, the Neutrosophic fuzzy travelling salesman problem is given by

	A	B	C	D
A	∞	(3,4,5)	(2,3,4)	(4,5,6)
B	(2,4,6)	∞	(5,7,9)	(5,6,7)
C	(4,6,8)	(5,6,7)	∞	(2,4,5)
D	(3,4,6)	(3,5,6)	(4,6,8)	∞

Step 2: The evaluation matrix is given by

	A	B	C	D
A	∞	(0, 3.5)	(0, 2.5)	(0, 4.5
B	(- 0.5,3)	∞	(- 0.5,6)	(0, 5.5)
C	(- 0.5, 5)	(0, 5.5)	∞	(0, 3)
D	(- 0.5, 3.5)	(0, 4)	(- 0.5, 5)	∞

Step 3: The score functionof the matrix is given by

A B C D

A	∞ 7	5	9	
B	7	∞ 13	11	
C	11	11	∞	6
D	8	8	11	∞

Step 4: Find therowwise minimum element and write it on the right side

	A	B	C	D	
A	∞ 7	5	9 5		
B	7	∞ 13	11		7
C	11	11	∞	6	6
D	8	8	11	∞	8

Then divide each element of the matrix in rowwise. Thiswill create at least one ones in each row.

A	B	C	D	
A	∞ 1.4	1^N	1.8	5
B	1^N∞ 1.85	1.57	7	
C	1.83	1.83	∞1^N	6
D	$1^N 1^N$	1.375	∞	8

Step 5: Find thecolumnwise minimum element and write it below and then divide each element of the matrix in columnwise . Thiswill create at least one ones in each column.

The same matrix iscolumnwise. So, the resultant matrix is given by

	A	B	C	D	
A	∞ 1.4	[1^N]	1.8 5		
B	[1^N]	∞ 1.85	1.57	7	
C	1.83	1.83	∞	[1^N]	6
D	1^N[1^N]	1.375	∞	8	

Step 6: City A assigns to City C Distance 5
City B assigns to City ADistance 7
City C assigns to City D Distance 6
City D assigns to City B Distance 8

Therefore, the root conditions aresatisfied and is given by

A →C , B→A , C → D , D → B

A → C → D → B → A

TheNeutrosophicCrisp optimum solution is given by 5 + 6 + 8 + 7 = 26

CONCLUION

In this paper , to find the optimal solution for the Neutrosophic fuzzy travelling salesman problem in Pentagonal fuzzy number and also compared with crisp travelling salesman problem. In future, we extend this method in generalized Neutrosophic fuzzy number.

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