Modelling of Commercial Banks Capitals Competition Dynamics

N.Arabov,
Samarkand State University, Uzbekistan.
D.Nasimov,
Academy of Public Administration, Uzbekistan.
H.Khuzhayorov,
Samarkand State University, Uzbekistan.
Christo Ananth*,
Samarkand State University, Uzbekistan.
T. AnanthKumar,
IFET College of Engineering, India.

Abstract---According to the observations in this paper, an existing mathematical model of banking capital dynamics should be tweaked. First-order ordinary differential equations with a "predator-pray" structure make up the model, and the indicators are competitive. Numerical realisations of the model are required to account for three distinct sets of initial parameter values. It is demonstrated that a wide range of banking capital dynamics can be produced by altering the starting parameters. One of the three options is selected, and the other two are eliminated. The model is generalized taking into account fractional derivatives of the bank indicators for time, reflecting the rate of their change. Based on numerical calculations, it is established that reduction of the order of derivatives from units leads to a delay of banking capital dynamics. It is shown, that the less the order of derivatives from the unit, the more delay of dynamics of indicators. In all analyzed variants indicators at large times reach their equilibrium values.

Keywords---Credit Resources, Deposits, Dynamics of the Banking Capital, Fractional Derivatives, Loan, Mathematical Model, Own Capital of the Bank.

I. Introduction

Business entities, enterprises, and entrepreneurs need capital to establish new high-tech manufacturing facilities, especially early in the project life cycle. Some businesses deposit any excess cash with the bank during the same period. Bank capital is "dynamic," meaning it can change over time as a result of this constant change. The modeling of fund demand and supply, risks and interbank competition, borrower-client late payments, changes in market conditions, and unfavorable economic processes at the local and global levels is difficult to accomplish. Loan officers consider the borrower's ability to repay the loan in full and on time when determining whether or not to extend a loan to him or her. Many changes occur during a project's implementation that cannot be considered when determining the feasibility of debtors. The mathematical model of commercial banks' credit and deposit operations should be dynamic, allowing for a time-series analysis of key financial and economic indicators. It has been said (Capinski and Zastawniak 2003; Bensoussan& Zhang 2009; Greene 2012).Everyone knows that bank lending carries risk. The credit rating of a debtor is an essential factor in determining credit risk. The volatility of credit ratings can impact a bank's ability to conduct normal business. Most mathematical models, credit risk management methods, and other applications assume that debtors' credit ratings transitional probabilities are constant and similar. It is also well-known that debtors' credit ratings change over time due to various factors. The variability of income over time is a popular indicator for assessing long-term financial risk (Lando 2004; Bielecki, Rutkowski 2007; Kwok 2008). When all else is equal, portfolio diversification can help reduce market volatility. So the total variance is the sum of each component of the portfolio. Diversification reduces overall income volatility while lowering risk. Banks should be cautious when engaging in risky activities. Banks today are much more than just deposit takers. It is necessary to recognise their full commercial status. The mathematical modelling of commercial bank financial transactions is becoming increasingly difficult due to increased risk. Banks want to make as much money as possible by giving customers loans (both individuals and legal entities). The client is concerned about reducing the amount of additional interest charged. As a result, the bank's and borrower's interests clash. Samarskii and Mikhailov's (2002; Brauer and Castillo-Chavez 2012; Hastings 2013) principles have been successfully applied in modeling biological and ecological systems, wage and employment changes, and other areas. Theoretically, such models can be built between any two competitors. These models can be simplified using Lotka-Volterra equations. The following authors were particularly interested in models like these when it came to market competition dynamics (Tseng et al. 2014; Cooper, Nakanishi 1988; Lakka et al. 2013; Michalakelis et al. 2012;

Marasco et al. 2016). Also, competition models can assess bank health and economic growth (Jayakumar et al. 2018). Competition models are used to study American bank deposit and loan dynamics (Sumarti et al. 2014; Ansori et al. 2019). These models have many uses, but they also have some flaws. (Marasco et al. 2016) expresses concern: For example, economic factors that affect market share dynamics are constant over time; b) the models are frequently not adequately connected to economic theory; and c) the effectiveness of proposed models is determined by estimations of model parameters that determine competition roles. Unfortunately, the data used to derive these estimates is scarce, and numerical methods such as genetic algorithms and other approaches are used. These models can predict a wide range of competitive economic processes. As stated by (Comes 2012), a bidirectional capital transfer from the Mother Bank to the Subsidiary Bank and vice versa is considered in a three-level Lotka-Volterra (TLVR) model. In the TLVR model, the banking sector's equilibrium is analysed using the Fokker-Planck-Kolmogorov stochastic equation solution. Lotka-Volterra models for n-level banking can be created from scratch (Marasco et al. 2016).

Many researchers are now interested in fractional differential equations because they can accurately describe various phenomena. These systems are studied using fractional-order differential equations. Fractional-order systems are preferred because they allow more model freedom. These evolution equations produce fractional Brownian motion. Non-Markovian despite the motions being Gaussian (Das et al. 2011). The authors solved the fractional-order Lotka–Volterra equations using a nonlinear analytical method called homotopy perturbation. To derive fractional Lotka–Volterra equations from classical ones, fractional derivatives are used instead of first-order time derivatives. The fractional predator-prey and rabies models are numerically solved in (Ahmed et al. 2007). The scientists claim the system is as stable as an integer-order system for fractional-order systems. The authors used powerful analytical methods like the homotopy perturbation method to approximate an analytical solution for the fractional-order Lotka–Volterra model. The Caputo-Fabrizio fractional derivative and fractional calculus are used to study a three-dimensional Lotka-Volterra differential equation (Khalighi et al., 2021). The nonlinear population model's fractional operator derivatives produce a variety of dynamical behaviors. [Traditional] Many generalized fractional Lotka–Volterra models are proposed (Elettreby et al. 2017; Owolabi 2021; Amirian et al. 2020). Fractional competitive models for commercial banks appeared in the 1990s. By Fatmawati et al., the Lotka-Volterra competition model's parameter values are estimated using the genetic algorithm method. Atangana-Baleanu and Caputo derivatives are used to study commercial and rural bank competition in 2019. To summarise, both model operators provide useful data at the fractional-order parameter's points of interest. Wang et al. (2019) also use a competition model to study bank data dynamics, with encouraging results. These papers look at fractional operators (Caputo and its variants) and integers in general (including the Atangana–Baleanu case). Among Baleanu's works, Atangana stands head and shoulders above the rest. Because the Caputo-Fabrizio operator is a fractional operator of the fractal-fractional operator, this issue affects both operators (Atangana et al., 2020). This study compares integer-order relevant results for accurate data to fractal and fractional order parameter values. A study found that the Caputo-Fabrizio derivative fractal-fractional outperforms the integer-order definition. This short analysis shows that fractional calculus began to be applied widely in the analysis of bank activity. Mathematical models with fractional derivatives describe the bank data more adequately. A mathematical model for banking data through with fractal-fractional operator in the sense of Caputo derivative is also presented by Li et al (2020). It was shown that with varying fractal and fractional orders of derivatives one can obtain the best fitting to the data.

In this paper, we consider a mathematical model describing the dynamics of commercial bank capital where various bank indicators have a competitive character. The model by its structure is similar to the three-level Lotka-Volterra (TLV) model. However, here process and its indicators are absolutely others. First, we give general information on mathematical models. Then a modified mathematical model for bank capital dynamics is developed and numerically analyzed. Further, the model is generalized using fractional derivatives. Based on the numerical analysis the influence of the order of fractional derivative on the dynamics of the banking capital is established. In conclusion, we summarize the results.

II. Mathematical Models

In the classical form, the Lotka-Volterra “pray-predator” model can be written in the following form [7, 9]

$$\frac{dx_1}{dt} = (a - bx_2)x_1,$$
$$\frac{dx_2}{dt} = (-c + d \cdot x_1)x_2, \qquad - \ 1$$

where$x_1(t), x_2(t) -$ prays and predators numbers, respectively, a, b, c, d constant coefficients**.** A mathematical model to describe the behaviour of deposit and loan volumes between two commercial banks can be written as (Ansori et al. 2019).

$$\frac{dx_1}{dt} = ax_1\left(1 - \frac{x_1}{K_D}\right) - bx_1, \frac{dx_2}{dt} = cx_2\left(1 - \frac{x_2}{K_L\frac{x_1}{K_D}}\right) - dx_2, \qquad (2)$$

where$x_1(t), x_2(t)$ are amounts of deposit and loan, respectively, a, b, c, d, K_L, K_D −constant parameters.

It is common to refer to banking micro-variables as a complex dynamic system in the banking industry because of their high variability. Deposits and loans (based on the transport equation) are two types of financial instruments: reserves and equity. Reserves and equity are two financial instruments (based on Selyutin and Rudenko, 2013). Stochastic differential equations (in the form of a probability distribution) [30] develop two systems of differential equations related to one another for the elements of a bank's balance sheet, which are then applied to the elements of the bank's balance sheet. In both systems, two variables are used: the deposit and the loan. Ansori et al. (2019) use a logistic model to compare the deposit and loan volumes of two different banks in their study. Savings and debt transfer are two aspects of four different models that are discussed further below. Depositors in bank 1 transfer their funds to bank 2, whereas borrowers transfer debts owed to bank 2 to bank 1. This is the model that is produced as a result.

$$\frac{dD_1}{dt} = g_{D1}D_1\left(1 - \frac{D_1}{K_{D1}}\right) - w_1D_1 - a_2D_1D_2, \frac{dL_1}{dt} = g_{L1}L_1\left(1 - \frac{L_1}{K_{L1}\frac{D_1}{K_{D1}}}\right) - b_1L_1 - c_2L_2L_1,$$

$$\frac{dD_2}{dt} = g_{D2}D_2\left(1 - \frac{D_2}{K_{D2}}\right) - w_2D_{2_1} + a_1D_1D_2, \frac{dL_2}{dt} = g_{L2}L_2\left(1 - \frac{L_2}{K_{L2}\frac{D_2}{K_{D2}}}\right) - b_2L_2 + c_2L_2L_1,$$

where D_i and L_i denote the volume of deposit and loan for the bank i , g_{Di}, K_{Di}, w_i, a_i, g_{Li}, K_{Li}, b_i, c_i are constant positive parameters, $i = 1; 2$.

Dynamics of maximum profit by commercial banks $(x_1(t))$ and rural banks $(x_2(t))$ in Indonesia in [27] is written in the form

$$\frac{dx_1}{dt} = a_1x_1\left(1 - \frac{x_1}{c_1}\right) - b_1x_1x_2, \frac{dx_2}{dt} = a_2x_2\left(1 - \frac{x_2}{c_2}\right) - b_2x_1x_2, \qquad (3)$$

where$a_1, a_2, b_1, b_2, c_1, c_2$ are constants.

Model (3) depicts the competition between commercial and rural banks in an environment where fractal-fractional operations are not used. By Atangana et al. (2020) the model (3) is generalized, where instead of $\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ the fractal-fractional Caputo-Fabrisio operators ${}^{CF}D_{o,t}^{u,v}(x_1(t))$ and ${}^{CF}D_{o,t}^{u,v}(x_2(t))$ are used.

The equilibrium state of the models (1), (2), (3) can be obtained by setting

$$\frac{dx_1}{dt} = 0, \frac{dx_2}{dt} = 0, {}^{CF}D_{o,t}^{u,v}(x_1(t)) = 0, {}^{CF}D_{o,t}^{u,v}(x_2(t)) = 0.$$

The TLV model that represents a tri-trophic capital chain between Mother bank, Subsidiary bank and individuals has the form [18]

$$\frac{dx_1}{dt} = x_1(t)\big(a_1 - b_1x_2(t) + c_1x_3(t)\big), \qquad (4)$$

$$\frac{dx_2}{dt} = x_2(t)\big(-a_2 + b_2x_1(t)\big), (5)$$

$$\frac{dx_3}{dt} = x_3(t)\big(a_3 - b_3x_1(t)\big), (6)$$

where$x_1(t), x_2(t), x_3(t)$ is the number of Mother bank, Subsidiary bank and Individuals, respectively, $a_i, b_i, c_1, i = 1,2,3$ are positive constants.

Some other approach to modelling of dynamics of the banking capital is given in [32]. The model is presented as

$$\frac{\partial D}{\partial t} = (r_D - a_{11})D + a_{12}C + a_{13}r_DY(t), \qquad (7)$$

$$\frac{dK}{dt} = -a_{21}K + a_{22}C + \frac{a_{23}}{r_K(t)}Y'(t) (8)$$

$$\frac{dC}{dt} = a_{31}(r_KK - r_DD) - a_{32}C, (9)$$

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where $D(t)$ - the volume of deposits of clients at the moment t, $K(t)$ - size of a credit portfolio of the bank at the moment t, $C(t)$ - own capital of the bank which has been saved up by some moment t, $Y(t)$ - the aggregate income of clients of the bank at the moment t in annual expression, r_D, r_K - interest rates under deposits and bank credits, respectively, $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{31}, a_{32}$ - positive constants.

In the models (4) - (6) and (7) - (9) three spices are used in conjunction with one another. We can use these models to solve ordinary differential equations, but first, we must ensure that the solution is stable. Changing the model parameters can achieve an equilibrium state between three different spices in the presence of a set of model constants. This article also discusses the model parameters that affect the solution's stability. The stability of the bank's differential equation model solution systems is inversely proportional. At the equilibrium points of their respective models of equations, (4) to (6) and (7) to (9) respectively, are the steady solutions to the system of equations.

When the models (7) - (9) are examined in detail, it is discovered that a stable solution to the system of equations can only be found within a very narrow range of the model parameters. A variant of this model, which incorporates some additional features, is presented here due to this research. This assumption was made when formulating equation (8), which states that the rate at which the credit portfolio dynamics changes is influenced by the credit policy of the financial institution. Possessing large amounts of own capital on a broad basis allows financial institutions to broaden the scope of their credit portfolios while also increasing the rate at which their credit portfolios grow (Vlasenko 2013). Addition of the member a_{22} C to the right-hand side of equation (8) was used to take into consideration the given circumstance. Another assumption is being used in this situation. We believe that the volume of involved capital from investors, i.e. deposits, as a component of the bank's financial resources has an impact on the increase in the dynamics of a credit portfolio's growth. Because of this assumption, we replace $a_{22}C$ on $a_{22}D$ in (8). Thus, equation (8) takes the form

$$\frac{dK}{dt} = a_{22}D - a_{21}K + \frac{a_{23}}{r_K(t)}Y'(t). \quad (10)$$

The equations (7) and (9) do not change. Then the system of equations (7) - (9) with taking into account (10) we write in the following form

$$\frac{\partial D}{\partial t} = (r_D - a_{11})D + a_{12}C + a_{13}r_D Y(t), \quad (11)$$

$$\frac{dK}{dt} = a_{22}D - a_{21}K + \frac{a_{23}}{r_K(t)}Y'(t), \quad (12)$$

$$\frac{dC}{dt} = a_{31}(r_K K - r_D D) - a_{32}C. \quad (13)$$

(Atangana et al. 2020; Li et al. 2020) have recently begun to use fractional derivatives in analyzing bank activity models. They are considered more general and realistic than models with integer-order derivatives because they can describe memory and heredity processes. Based on these works, we propose the generalization of models (4)-(6) and (11), (12) and (13).

a) Model (4) – (6)

$$\frac{d^\alpha x_1}{dt^\alpha} = x_1(t)\big(a_1 - b_1x_2(t) + c_1x_3(t)\big), \quad (14)$$

$$\frac{d^\beta x_2}{dt^\beta} = x_2(t)\big(-a_2 + b_2x_1(t)\big), \quad (15)$$

$$\frac{d^\gamma x_3}{dt^\gamma} = x_3(t)\big(a_3 - b_3x_1(t)\big), \quad (16)$$

b) Model (11) – (13)

$$\frac{d^\alpha D}{dt^\alpha} = (r_D - a_{11})D + a_{12}C + a_{13}r_D Y, \quad (17)$$

$$\frac{d^\beta K}{dt^\beta} = a_{22}D - a_{21}K + \frac{a_{23}}{r_K(t)}Y'(t). \quad (18)$$

$$\frac{d^\gamma C}{dt^\gamma} = a_{31}(r_K K - r_D D) - a_{32}C, \quad (19)$$

where$\alpha, \beta, \gamma$is the order of derivatives.

The system of the ordinary differential equations (11) - (13) in the matrix form we write as

$$\frac{dX}{dt} = AX + B, \quad (20)$$

where$X = \begin{pmatrix} D \\ K \\ C \end{pmatrix}, A = \begin{pmatrix} r_D - a_{11} 0 a_{12} \\ a_{22} - a_{21} 0 \\ -r_D a_{31} a_{31} r_K - a_{32} \end{pmatrix}, B = \begin{pmatrix} a_{13} r_D Y(t) \\ \frac{a_{23}}{r_K(t)} Y'(t) \\ 0 \end{pmatrix}$.

To solve (20) it is necessary to set the initial conditions $X(0) = X_0$, where$X_0 = (D_0 K_0 C_0)^T$, $D_0, K_0, C_0$are initial values of $D, K \rightleftarrows, C$, i.e.$D(0) = D_0, K(0) = K_0 C(0) = C_0$, T is a transposition sign.

The equilibrium state of the system (20) is defined as a stationary solution $\frac{dX}{dt} = 0$, that gives a system of linear algebraic equations $AX = -B$with respect to $D, K \rightleftarrows, C$. The solution of the last system of equations at $Y(t) = Y_0 = const$is

$$D^0 = a_{13} a_{32} a_{21} r_D Y_0 / d, K^0 = a_{13} a_{32} a_{22} r_D Y_0 / d, C^0 = a_{13} a_{31} r_D (a_{22} r_K - a_{21} r_D) Y_0 / d, \qquad (21)$$

where $d = a_{13} a_{12} a_{21} r_D + a_{11} a_{32} a_{21} - a_{31} a_{12} a_{22} r_K - a_{32} a_{21} r_D$.

The system (12) solution must be stable for the equilibrium state to exist. The eigenvalues of matrix A will be investigated in more detail later on. At least one of the three eigenvalues should be complex-conjugated with negative real parts, and the other two should be positive real parts. It satisfies the Raus-Gurvits requirements (Walter 1998). Fractional derivatives of the Riemann-Liouville, Grünwald-Letnikov, Caputo, Caputo-Fabrizio, Atangana-Baleanu, and other functions can be used.

III. Numerical Analysis of Models

Here we give the numerical analysis of the system of ordinary differential equations describing the dynamics of the banking capital. At first, the system of equations with integer derivatives is analyzed. Then we give a short information on fractional-order derivatives. At last, we numerically analyze the system of ordinary differential equations of fractional order (17) - (19). In order to begin, we must first solve the system of equations (11) - (13). This system is solved by using a standard library of the programming language Matlab, which is available online. Figure 1 depicts some of the outcomes. Table 1 shows the calculated values of parameters at their initial and intermediate stages.

3.1. Numerical Analysis of the System of the Equation of Integer Order

Table 1: The Initial and Intermediate Calculated Values of Parameters for the Problem

Parameters, units	Variant 1, Fig.1a	Variant 2, Fig.1b	Variant 3, Fig.1c
$r_D, 1/year$	0,125	0,103	0,139
$r_K, 1/year$	0,234	0,225	0,236
$a_{11}, 1/year$	0,139	0,247	0,225
$a_{12}, 1/year$	0,00001	0,00001	0,01
$a_{13}, 1/year$	0,219	0,263	0,104
$a_{21}, 1/year$	0,212	0,209	0,104
$a_{22}, 1/year$	0,195	0,192	0,149
$a_{23}, 1/year$	0	0	0
$a_{31}, 1/year$	0,474	0,493	0,351
$a_{32}, 1/year$	0,101	0,102	0,108
$\lambda_1, 1/year$	-0,014	-0,209	0,123+0,045i
$\lambda_2, 1/year$	-0,212	-0,143	-0,123- 0,045i
$\lambda_3, 1/year$	-0,101	-0,102	-0,052
Y_0,bln$	150	150	150
D_0,bln$	350	20	50
K_0,bln$	200	10	40
C_0,bln$	150	30	30
D^0,bln$	297,146	28,487	27,271
K^0,bln$	273,315	26,195	39,173
C^0,bln$	124,777	14,326	17,845

Figure 1 depicts the relationship between various financial indicators and the initial data. Some indicators are both decreasing and increasing in a monotonous fashion. The local minimum and maximum values break up the data's monotony. The values of the initial parameters impact the nature of the dependencies as well (see Figure 1).

Figure 1b illustrates this by showing that, while D(t) in Figures 1a and 1c is monotonically decreasing, it is increasing in Figure 1. In Figures 1a and 1c, non-monotonic K(t) dynamics are shown, whereas in Figure 1, monotonically increasing K(t) dynamics are shown. In Figure 1c, the eigenvalues of matrix A are represented by one real and two complex-conjugated eigenvalues. Figure 1c shows the real Eigenvalues of Matrix A, as received. Expect the oscillatory dynamics of financial indicators in this environment to continue. As shown in Table 1, the fluctuations of the indicators are not well expressed and are only apparent for indicator C due to the small values of the imaginary parts of eigenvalues (t). Matrix A has eigenvalues that can be described as either all negative real or as two complex-conjugated with negative real parts and one real negative (as shown in Figures 1a and b), depending on the initial parameter variations (Fig 1 c). K0/D0, the bank's credit portfolio ratio to client investors' deposits, is an essential financial indicator to watch when the bank is in equilibrium. The liquidity of a commercial bank's balance sheet can be gauged using this coefficient (Vlasenko 2013). The higher the value, the more difficult it is for the bank to serve its customers due to the lack of liquidity. This indicates the bank's inefficiency in financial and economic activity, reflected in its low profitability. As we investigate each parameter, we will calculate this factor (Tab. 1). For the variant 1,2 with Fig. 1a,b we have K^0/D^0=0,92, and for the variant 3 with Fig.1c: K^0/D^0=1,44. The above critical values cannot be compared to these. Financially, the bank prefers Variant 3 with Fig.1c.

In the model, C(t) represents only the capitalized profit, which is the factor a31 considers. Growing crediting dynamics K(t) does not always mean growing crediting dynamics C(t). This analysis excludes time-revalued basic capital and additional investor investments but not credit-depositary commercial activity (e.g., a bank's loan portfolio). All of this clearly reflected Dynamics C. (t). Inflation-driven revaluation of basic capitals requires consideration of financial and economic indicators.

3.2. Fractional Derivatives

There are many definitions of fractional-order derivatives. Detailed information on this topic can be found in (Podlubny 1999; Samko et al. 1993; Yang 2019). Here we give some definitions of the most used fractional derivatives. The Grünwald-Letnikov derivative of the function $f(t)$is defined as,

$$D^{\alpha} f(t) = \lim_{h\to\infty} \frac{1}{h^{\alpha}} \sum_{0\le r<m} (-1)^r \binom{\alpha}{r} f(t+(\alpha-r)h), \qquad (22)$$

whereα is the order of derivative, m is the smallest natural number such that

$$p \le m,\ r \in N, \binom{\alpha}{r} = \frac{p(p-1)\cdots(p-r+1)}{r(r-1)\cdots 1}. \qquad (23)$$

The Riemann-Liouville fractional derivative of order α is given by

$${}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau)d\tau, n-1<\alpha<n$$

where$\Gamma(\cdot)$ is the gamma function.

An alternative differentiation operator to the Grünwald-Letnikov and Riemann-Liouville operators was proposed by Caputo,

$$D^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{dt^n} d\tau, n-1<\alpha<n, n=[\alpha]+1. \qquad (24)$$

Caputo's operator has some advantages over the Riemann-Liouville operator: For example, the Riemann-Liouville fractional derivative's Laplace transform yields boundary conditions containing the lower and upper limits of the derivative. These situations are difficult to describe physically. The Caputo derivative's Laplace transform yields integer order boundary conditions. As an extra, the Caputo-Riemann-Liouville derivative of a constant is zero while it is not. We also use the Caputo derivative for its advantages. Because, we will be solving the equations using the finite-difference method, we will briefly discuss fractional derivative approximations.

3.3. Numerical Analysis of the System of the Equation of Fractional Order

To solve (17) – (19) we also use the finite difference method, where for the discretization of the fractional derivatives a numerical integration of the Caputo representation will be used. So, we have the following approximation in the case α≤1,n=1,a=0 (Liu et al. 2006; Sweilam et al. 2012; Xia et al. 2009).

$$\frac{d^{\alpha} f(t_i)}{dt^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \int_a^{t_i} (t_i-s)^{n-\alpha-1} \frac{df(s)}{dt} ds = \frac{1}{\Gamma(1-\alpha)} \sum_{j=1}^{i} \int_{(j-1)h}^{jh} \left[\frac{f_j-f_{j-1}}{h} + o(h)\right] (ih-s)^{-\alpha} ds =$$

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$$= \frac{1}{\Gamma(2-\alpha)h^{\alpha}} \sum_{j=1}^{i} \int_{(j-1)h}^{jh} \left(f_j - f_{j-1}\right)[(i-j+1)^{1-\alpha} - (i-j^{1-\alpha})] +$$
$$+ \frac{1}{\Gamma(2-\alpha)} \sum_{j=1}^{i} \int_{(j-1)h}^{jh} [(i-j+1)^{1-\alpha} - (i-j^{1-\alpha})] o(h^{2-\alpha}) = \quad (22)$$
$$= \sigma_{\alpha,h} \sum_{j=1}^{i} \omega_j^{\alpha} \left(f_{i-j+1} - f_{i-j}\right) + o(h),$$

where h is the mesh step, $\sigma_{\alpha,h} = \frac{1}{\Gamma(2-\alpha)h^{\alpha}}$, $\omega_j^{\alpha} = j^{1-\alpha} - (j-1)^{1-\alpha}$.

The equations (17) - (19) with use (22) were approximate in the following form

$$\frac{\tau^{1-\alpha}}{\Gamma(2-\alpha)} \left[\sum_{k=0}^{i-1} \frac{D_{k+1} - D_k}{\tau} ((i-k+1)^{1-\alpha} - (i-k)^{1-\alpha}) + \frac{D_{i+1} - D_i}{\tau} \right] =$$
$$= (r_D - a_{11}) D_i + a_{12} C_i + a_{13} r_D Y_i, \quad (23)$$
$$\frac{\tau^{1-\beta}}{\Gamma(2-\beta)} \left[\sum_{k=0}^{i-1} \frac{K_{k+1} - K_k}{\tau} ((i-k+1)^{1-\beta} - (i-k)^{1-\beta}) + \frac{K_{i+1} - K_i}{\tau} \right] =$$
$$= a_{22} D_i - a_{21} K_i + \frac{a_{23}}{r_K} Y_i', \quad (24)$$
$$\frac{\tau^{1-\gamma}}{\Gamma(2-\gamma)} \left[\sum_{k=0}^{i-1} \frac{C_{k+1} - C_k}{\tau} ((i-k+1)^{1-\gamma} - (i-k)^{1-\gamma}) + \frac{C_{i+1} - C_i}{\tau} \right] =$$
$$= a_{31} (r_K K_i - r_D D_i) - a_{32} C_i, \quad (25)$$

where D_i K_i,$C_i$are grid functions in a time point t_i, corresponding to $D(t)$,$K(t)$, $C(t)$, respectively, τ is the time step.

From the explicit finite difference schemes (23) - (25) we define D_{i+1},K_{i+1},C_{i+1}

$$D_{i+1} = \left((r_D - a_{11})\Gamma(2-\alpha)\tau^{\alpha} + 1\right) D_i + (a_{12} C_i + a_{13} r_D Y) \Gamma(2-\alpha)\tau^{\alpha} -$$
$$- \left[\sum_{k=0}^{i-1} \frac{D_{k+1} - D_k}{\tau} ((i-k+1)^{1-\alpha} - (i-k)^{1-\alpha}) + \frac{D_{i+1} - D_i}{\tau} \right], \quad (26)$$
$$K_{i+1} = \left(1 - a_{21}\Gamma(2-\beta)\tau^{\beta}\right) K_i + \left(a_{22} D_i + a_{23} Y_i'/r_K\right) \Gamma(2-\beta)\tau^{\beta} -$$
$$- \left[\sum_{k=0}^{i-1} \frac{K_{k+1} - K_k}{\tau} \left((i-k+1)^{1-\beta} - (i-k)^{1-\alpha}\right) + \frac{K_{i+1} - K_i}{\tau} \right], \quad (27)$$
$$C_{i+1} = (1 - a_{32}\Gamma(2-\gamma)\tau^{\gamma}) C_i + a_{31}(r_K K_i - r_D D_i) \Gamma(2-\gamma)\tau^{\gamma} -$$
$$- \left[\sum_{k=0}^{i-1} \frac{C_{k+1} - C_k}{\tau} ((i-k+1)^{1-\gamma} - (i-k)^{1-\gamma}) + \frac{C_{i+1} - C_i}{\tau} \right]. \quad (28)$$

The relations (26) - (28) define solutions on the time layer t_{j+1} based on solutions on the previous layers.

According to (26)-(28), numerical calculations were carried out for those three variants, the parameters of which are given in Table 1. Some results are shown in Fig. 2-4. The orders of fractional derivativesα, β, γ were chosen in different ways with a sequential decrease from 1. In Fig. 2-4 for comparison with the classical case, the graphs from Fig. 1 are also shown. In Fig. 1 a,b,c the parameters α, β, γ alternately take the value 0.9, while the rest are equal to one. There is slight lagging dynamics on the graphs. In fig. 2 d,e,f parameters, alternately take values of 0.8, while the rest have values of 0.9. The decrease in the parameter values from unity, in this case, is more significant than in Fig. 2 a,b,c. In this case, a more significant delay in dynamics of D,K,C is observed. Consequently, a decrease in the order of fractional derivatives in the model (17) - (19) leads to lagging dynamics of all indicators D,K,C. In this case, the smaller the values α, β, γ from unity, the more pronounced the phenomenon of delay. With increasing time, the retarded dynamics of D,K,Cweakens and asymptotically go to the stationary values determined for the model with integer derivatives. Consequently, the indicators$D$,K,C reach the same equilibrium values, only with a certain delay, the magnitude and duration of which is determined by the values α, β, γ. The calculation results for options 2 and 3 are shown in Fig. 3,4 for the same sets of parameters α, β, γ. For these variants, similar results were obtained as in the first variant. Here, a decrease in the values of parameters α, β, γ from unity leads to the same lagging dynamics, although the dynamics of indicators D,K,C is completely different here.

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IV. Conclusion

In this paper, a modified mathematical model of the dynamics of the capital of a commercial bank is developed. A numerical calculation was carried out for three variants of initial parameters, for which the dynamics of various indicators of the bank's activity is shown, such as D(t) - the volume of deposits of clients at the moment t, K(t) - the size of a credit portfolio of the bank at the moment t, C(t) - own capital of the bank which has been saved up by some moment t. It is shown, that depending on the values of the initial parameters, it is possible to obtain different dynamics of these indicators. Further, the model is generalized using fractional derivatives of the above indicators, reflecting their rate of change over time. Based on the numerical analyses of the model, the role of the order of fractional derivatives on the dynamics of bank capital is estimated. It is shown, that a decrease in the order of the derivatives from unity leads to lagging dynamics of indicators. In this case, the more the orders of the derivatives decrease from unity, the more pronounced the delay effect is. In all the variants considered, over time, the indicators of bank capital reach their equilibrium values, which are the same for both the model with integer derivatives and the model with fractional derivatives.

References

[1] Capinski, M., &Zastawniak, T. (2003). Mathematics for finance. An Introduction, – 321p.
[2] Bensoussan, A., & Zhang, Q. (2009). (Guest editors). Mathematical modeling and numerical methods in finance. Elsevier.
[3] Greene, W.H. (2000). Econometric analysis 4th edition. International edition, New Jersey: Prentice Hall.
[4] Lando, D. (2009). Credit risk modeling: theory and applications. Princeton University Press.
[5] Bielecki, T.R., Jeanblanc, M., &Rutkowski, M. (2007). Introduction to mathematics of credit risk modeling. Stochastic models in mathematical finance, CIMPA–UNESCO–MOROCCO school, 1-78.
[6] Kwok, Y.K. (2008). Mathematical models of financial derivatives. Springer.
[7] Samarskii, A.A., & Mikhailov, A.P. (2001). Principles of mathematical modelling: Ideas, methods, examples. CRC Press.
[8] Brauer, F., Castillo-Chavez, C., & Castillo-Chavez, C. (2012). Mathematical models in population biology and epidemiology (Vol. 2, p. 508). New York: Springer.
[9] Hastings, A. (Ed.). (2013). Population biology: concepts and models. Springer Science & Business Media.
[10] Tseng, F.M., Liu, Y.L., & Wu, H.H. (2014). Market penetration among competitive innovation products: The case of the Smartphone Operating System. Journal of Engineering and Technology Management, 32, 40-59.
[11] Cooper, L.G., & Nakanishi, M. (1989). Market-share analysis: Evaluating competitive marketing effectiveness (Vol. 1). Springer Science & Business Media.
[12] Lakka, S., Michalakelis, C., Varoutas, D., & Martakos, D. (2013). Competitive dynamics in the operating systems market: Modeling and policy implications. Technological Forecasting and Social Change, 80(1), 88-105.
[13] Michalakelis, C., Christodoulos, C., Varoutas, D., & Sphicopoulos, T. (2012). Dynamic estimation of markets exhibiting a prey–predator behavior. Expert Systems with Applications, 39(9), 7690-7700.
[14] Marasco, A., Picucci, A., & Romano, A. (2016). Market share dynamics using Lotka–Volterra models. Technological forecasting and social change, 105, 49-62.
[15] Jayakumar, M., Pradhan, R.P., Dash, S., Maradana, R.P., & Gaurav, K. (2018). Banking competition, banking stability, and economic growth: Are feedback effects at work?. Journal of Economics and Business, 96, 15-41.
[16] Sumarti, N., Nurfitriyana, R., &Nurwenda, W. (2014, February). A dynamical system of deposit and loan volumes based on the Lotka-Volterra model. In AIP Conference Proceedings (Vol. 1587, No. 1, pp. 92-94). American Institute of Physics.
[17] Ansori, M.F., Sidarto, K.A., & Sumarti, N. (2019). Model of deposit and loan of a bank using spiral optimization algorithm. J. Indones. Math. Soc, 25, 292-301.
[18] Comes, C.A. (2012). Banking system: three level Lotka-Volterra model. Procedia Economics and Finance, 3, 251-255.
[19] Das, S., & Gupta, P.K. (2011). A mathematical model on fractional Lotka–Volterra equations. Journal of Theoretical Biology, 277(1), 1-6.
[20] Ahmed, E., El-Sayed, A.M.A., & El-Saka, H.A. (2007). Equilibrium points, stability and numerical solutions of fractional-order predator–prey and rabies models. Journal of Mathematical Analysis and Applications, 325(1), 542-553..

[21] Khalighi, M., Eftekhari, L., Hosseinpour, S., & Lahti, L. (2021). Three-species Lotka-Volterra model with respect to Caputo and Caputo-Fabrizio fractional operators. Symmetry, 13(3), 368.

[22] Elettreby, M.F., Al-Raezah, A.A., & Nabil, T. (2017). Fractional-order model of two-prey one-predator system. Mathematical Problems in Engineering, 2017.

[23] Owolabi, K. M. (2021). Numerical approach to chaotic pattern formation in diffusive predator–prey system with Caputo fractional operator. Numerical Methods for Partial Differential Equations, 37(1), 131-151.

[24] Amirian, M.M., Towers, I.N., Jovanoski, Z., & Irwin, A.J. (2020). Memory and mutualism in species sustainability: A time-fractional Lotka-Volterra model with harvesting. Heliyon, 6(9), e04816.

[25] Khan, M.A., Azizah, M., & Ullah, S. (2019). A fractional model for the dynamics of competition between commercial and rural banks in Indonesia. Chaos, Solitons & Fractals, 122, 32-46.

[26] Wang, W., Khan, M.A., Kumam, P., & Thounthong, P. (2019). A comparison study of bank data in fractional calculus. Chaos, Solitons & Fractals, 126, 369-384.

[27] Atangana, A., & Khan, M.A. (2020). Modeling and analysis of competition model of bank data with fractal-fractional Caputo-Fabrizio operator. Alexandria Engineering Journal, 59(4), 1985-1998.

[28] Li, Z., Liu, Z., & Khan, M.A. (2020). Fractional investigation of bank data with fractal-fractional Caputo derivative. Chaos, Solitons & Fractals, 131, 109528.

[29] Selyutin, V.V., & Rudenko, M.A. (2013). Mathematical model of the banking firm as tool for analysis, management and learning. Інформаційні технології в освіті, (16), 170-177.

[30] Sumarti, N., Fadhlurrahman, A., & Widyani, H.R. (2018). The Dynamical System of the Deposit and Loan Volumes of a Commercial Bank Containing Interbank Lending and Saving Factors. Southeast Asian Bulletin of Mathematics, 42(5).

[31] Ansori, M. F., Sidarto, K.A., & Sumarti, N. (2019, December). Logistic models of deposit and loan between two banks with saving and debt transfer factors. In AIP Conference Proceedings (Vol. 2192, No. 1, p. 060002). AIP Publishing LLC.

[32] Vlasenko M. (2013) A mathematical model of a conditional bank. Banking bulletin. Studzen. 20-25.

[33] Walter, W. (1998). Linear Differential Equations. In Ordinary Differential Equations (pp. 159-212). Springer, New York, NY.

[34] Podlubny I. (1999) Fractional Differential Equations. Academic Press, New York.

[35] Samko S., Kilbas A., Marichev O. (1993) Fractional Integrals and derivatives: Theory and Applications. Gordon and Breach, London.

[36] Yang, X.J. (2019). General fractional derivatives: theory, methods and applications. CRC Press..

[37] Liu, F., Zhuang, P., Anh, V., & Turner, I. (2005). A fractional-order implicit difference approximation for the space-time fractional diffusion equation. ANZIAM Journal, 47, C48-C68.

[38] Sweilam, N.H., Khader, M.M., & Mahdy, A.M.S. (2012). Numerical studies for solving fractional-order Logistic equation. International Journal of Pure and Applied Mathematics, 78(8), 1199-1210.