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A New Three Parameter Odds Generalized Lindley-Pareto Distribution

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Abstract
A new three parameter lifetime model, called Odds Generalized Lindley-Pareto distribution (OGLPD) is proposed for modeling lifetime data. A comprehensive account of the mathematical properties of the new distribution including estimation is presented. A data set has been analyzed to illustrate its applicability.

Keywords: Lindley distribution; Pareto distribution; Maximum likelihood estimation; Odds function; Transformed-Transformer family of distributions.

1. Introduction

Modeling of any real world phenomenon gets complicated. Statistical distributions are important for parametric inferences and also are commonly applied to describe real world phenomenon. Due to the usefulness of statistical distributions, their theories are widely studied and new distributions are developed. A number of methods have been developed to generate statistical distributions in the literature. Some methods are developed in the early days for generating univariate continuous distributions include methods based on differential equations developed by Pearson (1895), methods of translation developed by Johnson (1949), and methods based on quantile functions developed by Tukey(1960). At the end of twentieth century, McDonald (1984), Azzalini (1985), Marshall and Olkin (1997) proposed some general methods for generating a new family of distributions. In twenty first century, Eugene et al. (2002) proposed the beta-generated family of distributions, Jones (2009) and Cordeiro and de Castro (2011) extended the beta-generated family of distributions by using Kumaraswamy distribution in place of beta distribution. Alzaatreh et al. (2013) proposed a generalized family of distributions, called T-X (also called Transformed-Transformer) family, whose cumulative distribution function (cdf) is given by

$$F(x;\theta) = \int_a^{W[G(x)]} v(t)dt\,, \qquad (1.1)$$

where, the random variable $T \in [a,b]$, for $-\infty < a, b < \infty$ and $W[G(x)]$ be a function of the cdf $G(x)$ so that $W[G(x)]$ satisfies the following conditions:

(i) $W[G(x)] \in [a,b]$,
(ii) $W[G(x)]$ is differentiable and monotonically non-decreasing,
(iii) $W[G(x)] \to a$ as $x \to -\infty$ and $W[G(x)] \to b$ as $x \to \infty$.

I have defined a generalized class of any distribution having positive support. Taking $W(F_\theta(x)) = \dfrac{F_\theta(x)}{1-F_\theta(x)}$, the odds function, the cdf of the proposed generalized class of distribution is given by

$$F(x\,|\,\lambda,\theta) = \int_0^{\frac{F_\theta(x)}{1-F_\theta(x)}} f_\lambda(t)dt. \qquad (1.2)$$

In the present paper, we choose particular choice of $f_\lambda(t) = \dfrac{\lambda^2(1+t)}{1+\lambda}e^{-\lambda t}$ i.e. the Lindley distribution and $F_{a,\theta}(x) = 1-\left(\dfrac{a}{x}\right)^\theta$ i.e. Pareto distribution in (1.2). Hence, I call this distribution as Odds Generalized Lindley-Pareto distribution (OGLPD).

The paper is organized as follows. The new distribution is developed in section 2. A comprehensive account of mathematical properties including structural and reliability of the new distribution is provided in section 3. Maximum likelihood method of estimation of parameters of the distribution is discussed in section 4. A real life data set has been analyzed and compared with other fitted distributions with respect to Akaike Information Criterion (AIC) in section 5. Section 6 concludes.

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2. Formation of Odds Generalized Lindley-Pareto Distribution

The c.d.f. of the distribution is given by the form as

$$F(X;\lambda,\theta,a) = \int_0^{\left(\frac{x}{a}\right)^{\theta}-1} \frac{\lambda^2(1+x)}{1+\lambda} e^{-\lambda x} dx = 1 - \frac{1+\lambda\left(\frac{x}{a}\right)^{\theta}}{1+\lambda} e^{-\lambda\left(\left(\frac{x}{a}\right)^{\theta}-1\right)} \tag{2.3}$$

Also the p.d.f. of the distribution is given by

$$f(x;\lambda,\theta,\alpha) = \frac{\lambda^2\theta x^{2\theta-1}}{(1+\lambda)\alpha^{2\theta}} e^{-\lambda\left\{\left(\frac{x}{\alpha}\right)^{\theta}-1\right\}} \tag{2.4}$$

3. Statistical and Reliability Properties

3.1 Limit of the Probability Distribution Function

Since the c.d.f. of this distribution is $F(X) = 1 - \frac{1+\lambda\left(\frac{x}{a}\right)^{\theta}}{1+\lambda} e^{-\lambda\left(\left(\frac{x}{a}\right)^{\theta}-1\right)}$

So $\lim_{x\to a} F(X) = 0$ i.e. $F(a) = 0$

Now $\lim_{x\to\infty} F(X) = 1$ i.e. $F(\infty) = 1$

3.2 Descriptive Statistics of the Distribution

The mean of this distribution is as follows:

$$\mu_1' = E(X) = \frac{\lambda^2\theta e^{\lambda}}{(1+\lambda)a^{2\theta}} \int_a^{\infty} x^{2\theta} e^{-\lambda\left(\frac{x}{a}\right)^{\theta}} dx = \frac{a e^{\lambda}}{(1+\lambda)\lambda^{\frac{1}{\theta}}} \Gamma\left(\frac{1}{\theta}+2,\lambda\right)$$

The median of the distribution is calculated by the equation

$$1 - \frac{1+\lambda\left(\frac{M}{a}\right)^{\theta}}{(1+\lambda)} e^{-\lambda\left(\left(\frac{M}{a}\right)^{\theta}-1\right)} = \frac{1}{2}$$

The mode of the distribution is $a\left(\frac{2\theta-1}{\lambda\theta}\right)^{\frac{1}{\theta}}$

The r^{th} **order raw moment** of the distribution is as follows:

$$E(X^r) = \frac{\lambda^2\theta e^{\lambda}}{(1+\lambda)a^{2\theta}} \int_a^{\infty} x^{r+2\theta-1} e^{-\lambda\left(\frac{x}{a}\right)^{\theta}} dx = \frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}} \Gamma\left(\frac{r}{\theta}+2,\lambda\right) \tag{3.5}$$

Now putting suitable values of r in the above equation, I get Variance, Skewness, Kurtosis and Coefficients of variation of the Odds Generalized Lindley- Pareto Distribution (OGLPD).

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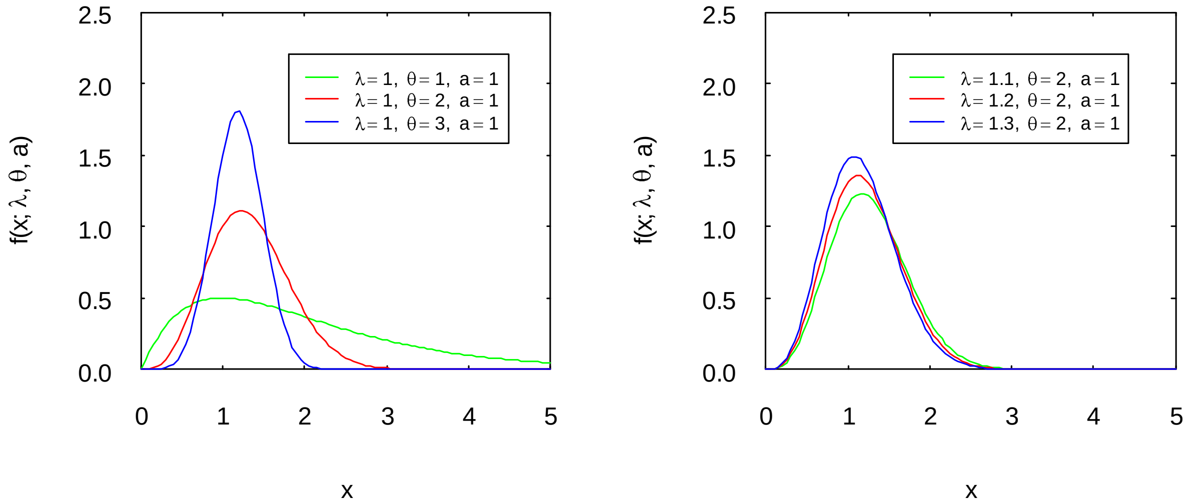

Figure 1: The probability density function of the OGLPD

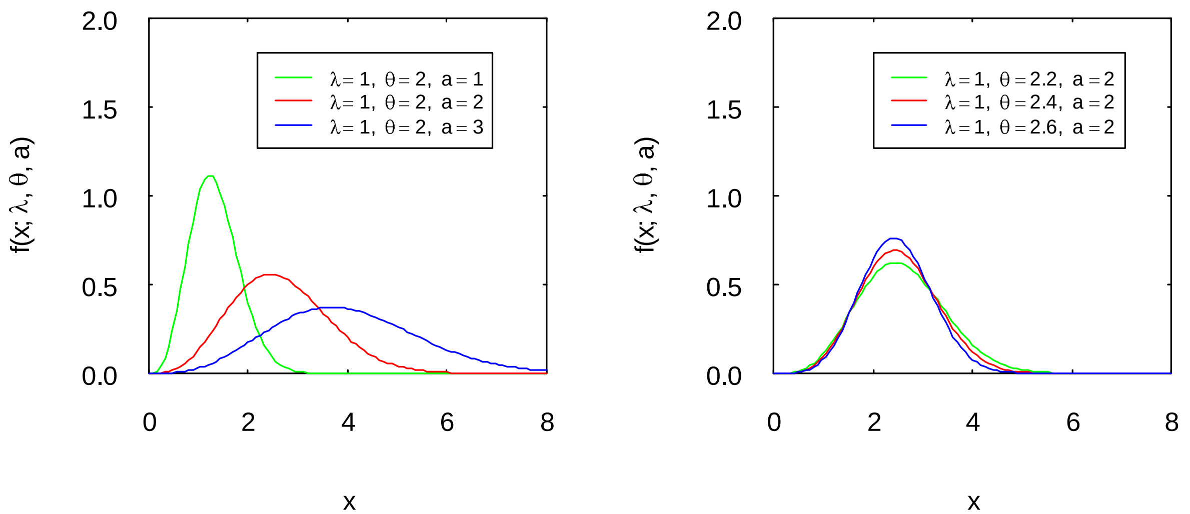

Figure 2: The probability density function of the OGLPD

Moment Generating Function (MGF):

$$M_X(t) = E(e^{tX}) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}} \Gamma\left(\frac{r}{\theta}+2, \lambda\right) \tag{3.6}$$

Characteristic Function (CF):

$$\Psi_X(t) = E(e^{itX}) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}} \Gamma\left(\frac{r}{\theta}+2, \lambda\right) \tag{3.7}$$

Cumulant Generating Function (CGF):

$$K_X(t) = \ln_e M_X(t) = \ln_e \left[\sum_{r=0}^{\infty} \frac{t^r}{r!} \frac{a^r e^{\lambda}}{(1+\lambda)\lambda^{\frac{r}{\theta}}} \Gamma\left(\frac{r}{\theta}+2, \lambda\right) \right] \tag{3.8}$$

Mean Deviation:

The mean deviation about the mean and the mean deviation about the median are defined by

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$$MD_{\mu}=2e^{\lambda}\left[\frac{a}{(1+\lambda)\lambda^{\frac{1}{\theta}}}\Gamma\left(\frac{1}{\theta}+2,\frac{\lambda\mu^{\theta}}{a^{\theta}}\right)-\mu\frac{1+\lambda\left(\frac{\mu}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left(\frac{\mu}{a}\right)^{\theta}}\right] \tag{3.9}$$

$$MD_{M}=-\mu+2\frac{ae^{\lambda}}{(1+\lambda)\lambda^{\frac{1}{\theta}}}\Gamma\left(\frac{1}{\theta}+2,\frac{\lambda\mu^{\theta}}{a^{\theta}}\right) \tag{3.10}$$

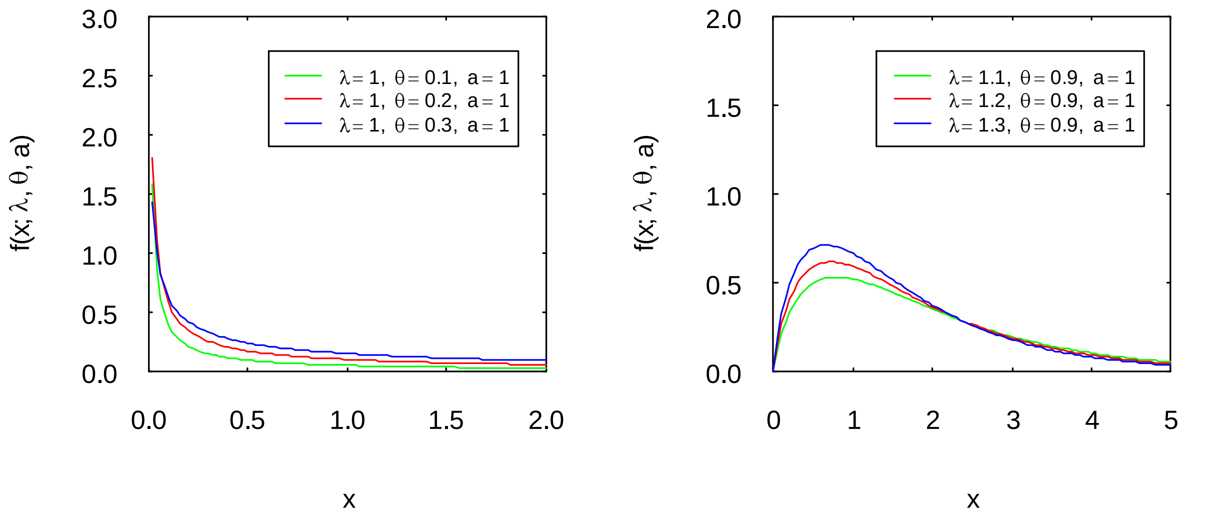

Figure 3: The probability density function of the OGLPD

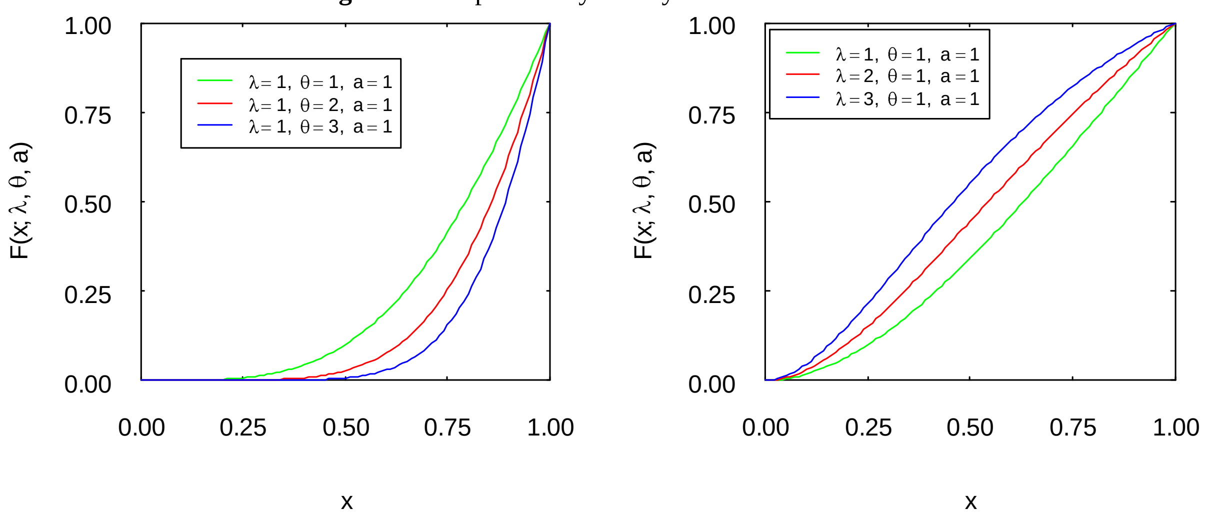

Figure 4: The cumulative distribution function of the distribution

Conditional Moments:

The residual life and the reversed residual life play an important role in reliability theory and other branches of statistics. Here, the r^{th} order raw moment of the residual life is given by

$$\mu_r^{'}(t)=E[(X-t)^r \mid X>t]=\frac{1}{\overline{F}(t)}\int_t^{\infty}(x-t)^r f(x)dx$$

$$= \frac{e^{\frac{\lambda t^{\theta}}{a^{\theta}}}}{\left[1+\frac{\lambda t^{\theta}}{a^{\theta}}\right]} \sum_{j=0}^{r} (-1)^{j} \binom{r}{j} t^{r-j} \frac{a^{j}}{\lambda^{\frac{j}{\theta}}} \Gamma\left(\frac{j}{\theta}+2, \frac{\lambda t^{\theta}}{a^{\theta}}\right)$$

The r[th] order raw moment of the reversed residual life is given by

$$m_r^{'}(t) = \frac{e^{\lambda}}{\left[(1+\lambda)-\left\{1+\lambda\left(\frac{t}{a}\right)^{\theta}\right\} e^{-\lambda\left\{\left(\frac{t}{a}\right)^{\theta}-1\right\}}\right]} \sum_{j=o}^{r} (-1)^{j} \binom{r}{j} t^{r-j} \frac{a^{j}}{\lambda^{\frac{j}{\theta}}} \left[\Gamma\left(\frac{j}{\theta}+2, \lambda\right)-\Gamma\left(\frac{j}{\theta}+2, \frac{\lambda t^{\theta}}{a^{\theta}}\right)\right]$$

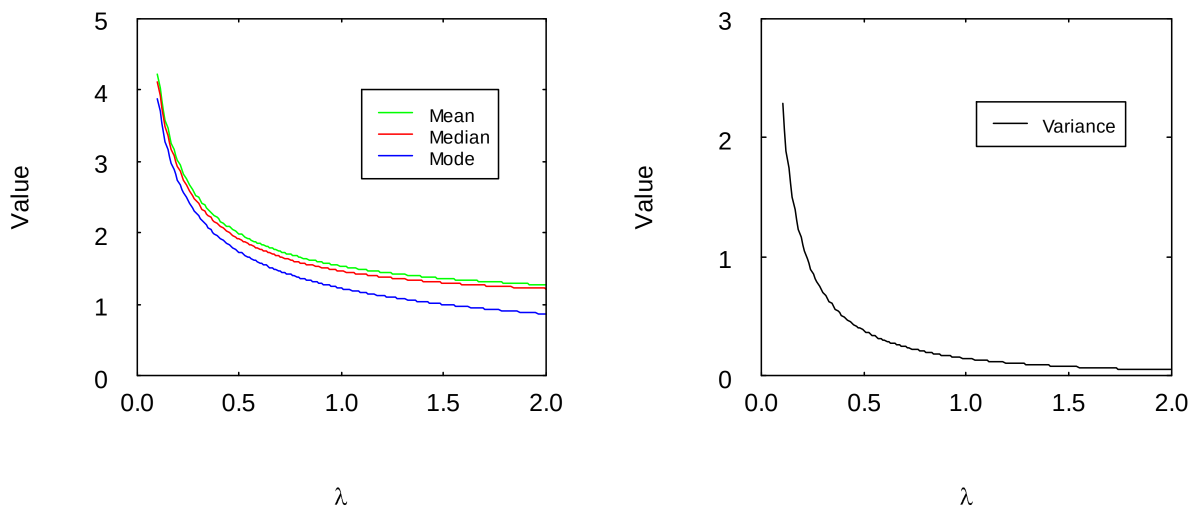

Figure 5: Mean, median, mode and variance of the distribution

L-Moments:

Define $X_{k:n}$ be the k[th] smallest moment in a sample of size n. The L-moments of X are defined by

$$\lambda_r = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^{k} \binom{r-1}{k} E\left[X_{r-k:r}\right], \quad r = 1,2,..$$

Now for OGLPD with parameter λ, θ and a, I have

$$E\left[X_{j:r}\right] = \frac{r!}{(j-1)!(r-j)!} \int_{a}^{\infty} x[F(x)]^{j-1}[1-F(x)]^{r-j} dF(x)$$

So the first four L- Moments are,

$$\lambda_1 = E\left[X_{1:1}\right] = \frac{a e^{\lambda}}{(1+\lambda)\lambda^{\frac{1}{\theta}}} \Gamma\left(\frac{1}{\theta}+2, \lambda\right)$$

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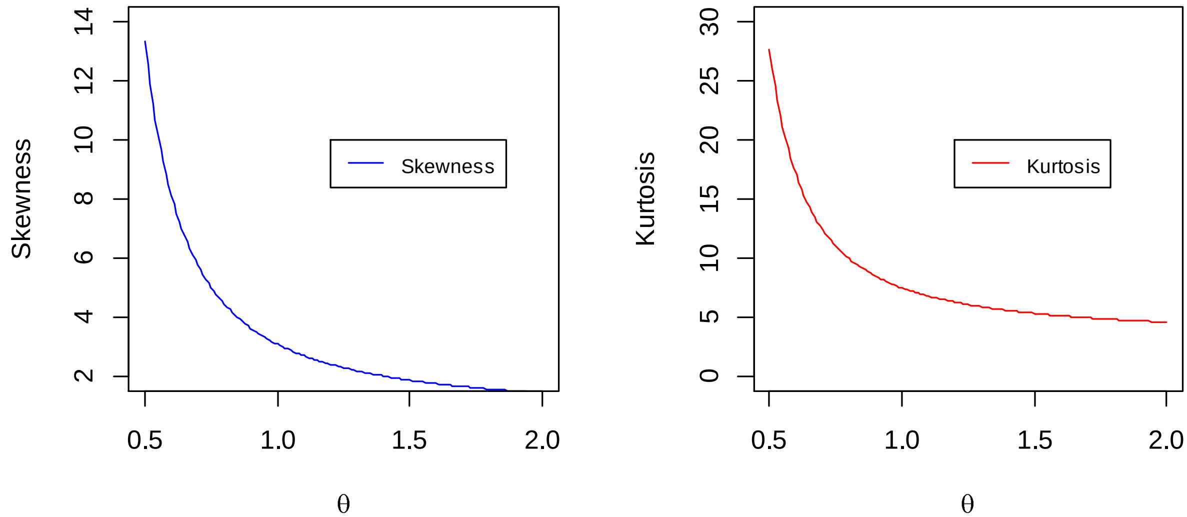
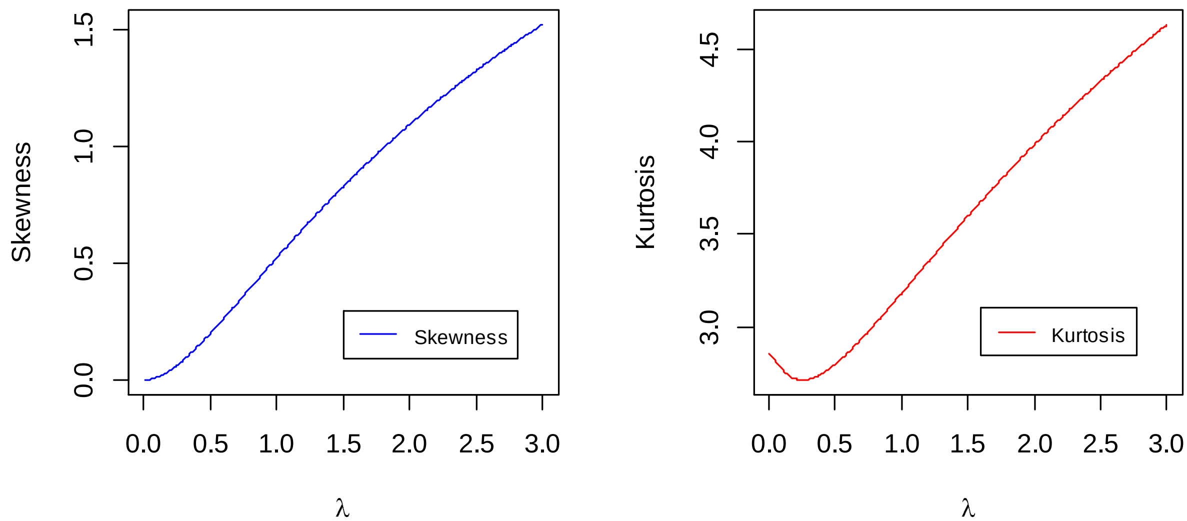


Figure 6: Skewness and Kurtosis of the OGLPD with different values of θ

Figure 7: Skewness and Kurtosis of the OGLPD with different values of λ



$$\lambda_3 = \frac{1}{3}E\left[X_{3:3} - 2X_{2:3} + X_{1:3}\right] = \frac{ae^{\lambda}\Gamma(\frac{1}{\theta}+2,\lambda)}{(1+\lambda)\lambda^{\frac{1}{\theta}}} - \frac{6ae^{2\lambda}}{(1+\lambda)^2\lambda^{\frac{1}{\theta}}}\left[\frac{\Gamma(\frac{1}{\theta}+2,2\lambda)}{2^{2+\frac{1}{\theta}}} + \frac{\Gamma(\frac{1}{\theta}+3,2\lambda)}{2^{3+\frac{1}{\theta}}}\right]$$

$$+\frac{6ae^{3\lambda}}{(1+\lambda)^3\lambda^{\frac{1}{\theta}}}\left[\frac{\Gamma(\frac{1}{\theta}+2,3\lambda)}{3^{2+\frac{1}{\theta}}} + \frac{2\Gamma(\frac{1}{\theta}+3,3\lambda)}{3^{3+\frac{1}{\theta}}} + \frac{\Gamma(\frac{1}{\theta}+4,3\lambda)}{3^{4+\frac{1}{\theta}}}\right]$$

$$\lambda_4=\frac{1}{4}E\left[X_{4:4}-3X_{3:4}+3X_{2:4}-X_{1:4}\right]=\frac{ae^{\lambda}\Gamma(\frac{1}{\theta}+2,\lambda)}{(1+\lambda)\lambda^{\frac{1}{\theta}}}-\frac{12ae^{2\lambda}}{(1+\lambda)^2\lambda^{\frac{1}{\theta}}}\left[\frac{\Gamma(\frac{1}{\theta}+2,2\lambda)}{2^{2+\frac{1}{\theta}}}+\frac{\Gamma(\frac{1}{\theta}+3,2\lambda)}{2^{3+\frac{1}{\theta}}}\right]$$

$$+\frac{30ae^{3\lambda}}{(1+\lambda)^3\lambda^{\frac{1}{\theta}}}\left[\frac{\Gamma(\frac{1}{\theta}+2,3\lambda)}{3^{2+\frac{1}{\theta}}}+\frac{2\Gamma(\frac{1}{\theta}+3,3\lambda)}{3^{3+\frac{1}{\theta}}}+\frac{\Gamma(\frac{1}{\theta}+4,3\lambda)}{34^{3+\frac{1}{\theta}}}\right]$$

$$-\frac{20ae^{4\lambda}}{(1+\lambda)^4\lambda^{\frac{1}{\theta}}}\left[\frac{\Gamma(\frac{1}{\theta}+2,4\lambda)}{4^{2+\frac{1}{\theta}}}-\frac{3\Gamma(\frac{1}{\theta}+3,4\lambda)}{4^{3+\frac{1}{\theta}}}+\frac{3\Gamma(\frac{1}{\theta}+4,4\lambda)}{4^{4+\frac{1}{\theta}}}-\frac{\Gamma(\frac{1}{\theta}+5,4\lambda)}{4^{5+\frac{1}{\theta}}}\right]$$

Quantile function:

Let X denotes a random variable with the probability density function 2.4. The quantile function, say Q(p), defined by F(Q(p)) = p is the root of the equation

$$1-\frac{1+\lambda\left(\frac{Q(p)}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left\{\left(\frac{Q(p)}{a}\right)^{\theta}-1\right\}}=p \tag{3.11}$$

3.3 Incomplete Moment, Bonferroni and Lorenz curves

The r[th] order incomplete moment of the OGLPD is

$$m_r^I(t)=\int_a^t x^r f(x)dx=\frac{e^{\lambda}a^r}{(1+\lambda)\lambda^{\frac{r}{\theta}}}\left[\Gamma\left(\frac{r}{\theta}+2,\lambda\right)-\Gamma\left(\frac{r}{\theta}+2,\lambda\left(\frac{t}{a}\right)^{\theta}\right)\right] \tag{3.12}$$

The Bonferroni and Lorenz curves are defined by

$$B(p)=\frac{m_1^I(x_p)}{p\mu} \tag{3.13}$$

and

$$L(p)=\frac{m_1^I(x_p)}{\mu} \tag{3.14}$$

Respectively, where μ=E(X) and $x_p=F^{-1}(p)$ which is to be calculated numerically using 3:10 for given p.

3.4 Order Statistics

Suppose X_1, X_2, X_3............X_n is a random sample from Eq.2.4. Let $X_{(1)}$, $X_{(2)}$, $X_{(3)}$................$X_{(n)}$, denote the corresponding order statistics. It is well known that the probability density function and the cumulative distribution function of the k[th] order statistic, say Y = $X_{(k)}$, are given by

$$f_Y(y)=\frac{n!}{(k-1)!(n-k)!}\frac{\lambda^2\theta y^{2\theta-1}}{a^{2\theta}}\left[1+\lambda\left(\frac{y}{a}\right)^{\theta}\right]^{n-k}\left[1-\frac{1+\lambda\left(\frac{y}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left(\left(\frac{y}{a}\right)^{\theta}-1\right)}\right]^{k-1}\left[\frac{e^{-\lambda\left(\left(\frac{y}{a}\right)^{\theta}-1\right)}}{1+\lambda}\right]^{n-k+1}$$

and (3.15)

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$$F_Y(y)=\sum_{j=k}^{n}\binom{n}{j}\left[1-\frac{1+\lambda\left(\frac{y}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left(\left(\frac{y}{a}\right)^{\theta}-1\right)}\right]^{j}\left[\frac{1+\lambda\left(\frac{y}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left(\left(\frac{y}{a}\right)^{\theta}-1\right)}\right]^{n-j} \quad (3.16)$$

3.5 Entropies

An entropy of a random variable X is a measure of variation of the uncertainty. A popular entropy measure is Renyi entropy (Renyi 1961). If X has the probability density function f(x), then Renyi entropy is defined by

$$H_R(\beta)=\frac{1}{1-\beta}\ln\left\{\int_a^{\infty} f^{\beta}(x)dx\right\} \quad (3.17)$$

$$H_R(\beta) = -\frac{\ln\lambda}{\theta}-\ln\theta+\ln\alpha+\frac{\lambda\beta}{1-\beta}-\frac{(2\beta-\frac{\beta}{\theta}+\frac{1}{\theta})}{1-\beta}\ln\beta + \frac{1}{1-\beta}\ln\Gamma\left(2\beta-\frac{\beta}{\theta}+\frac{1}{\theta},\lambda\beta\right)-\frac{\beta}{1-\beta}\ln(1+\lambda)$$

Shannon measure of entropy is defined as

$$H(f)=E[-\ln f(X)] = -\lambda-\ln\theta+\ln\alpha-\frac{\ln\lambda}{\theta}+\ln(1+\lambda)+\frac{e^{\lambda}}{1+\lambda}\Gamma(3,\lambda) - \frac{(2\theta-1)e^{\lambda}}{(1+\lambda)\theta}\Gamma^{(1)}(2,\lambda) \quad (3.18)$$

3.6 Reliability and related properties

The Reliability function is given by

$$R(x)=1-F(x)=\frac{1+\lambda\left(\frac{x}{a}\right)^{\theta}}{1+\lambda}e^{-\lambda\left(\left(\frac{x}{a}\right)^{\theta}-1\right)} \quad (3.19)$$

and the Hazard rate function is given by

$$r(t)=\frac{f(t)}{1-F(x)}=\frac{\lambda^2\theta}{a^{2\theta}\left[1+\lambda\left(\frac{t}{a}\right)^{\theta}\right]}t^{2\theta-1} \quad (3.20)$$

Now, $f(x)=\dfrac{\lambda^2\theta x^{2\theta-1}}{(1+\lambda)a^{2\theta}}e^{-\lambda\left\{\left(\frac{x}{a}\right)^{\theta}-1\right\}}$

i.e. $\ln f(x)=2\ln\lambda+\ln\theta+(2\theta-1)\ln x+\lambda-\lambda\left(\frac{x}{a}\right)^{\theta}-\ln(1+\lambda)-2\theta\ln a$

So $\dfrac{d}{dx}\ln f(x)=\dfrac{2\theta-1}{x}-\dfrac{\lambda\theta x^{\theta-1}}{a^{\theta}}$

$$\frac{d^2}{dx^2}\ln f(x)=-\frac{2\theta-1}{x^2}-\frac{\lambda\theta(\theta-1)x^{\theta-2}}{a^{\theta}}$$

For $\lambda > 0$, $\theta > 1$, $a > 0$ and $x > 0$, $\dfrac{d^2}{dx^2}\ln f(x)<0.$

So, the distribution is log-concave. Therefore, the distribution posses **Increasing failure rate** (IFR) and **Decreasing Mean Residual Life (DMRL)** property.

For $\lambda > 0$, $0 < \theta < 1$, $a > 0$ and $x > 0$, $\dfrac{d^2}{dx^2}\ln f(x)>0.$

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So, the distribution is log-convex. Therefore, the distribution posses **Decreasing failure rate (DFR)** and **Increasing Mean Residual Life (IMRL)** property.

Mean Residual Life (MRL) function is defined as

$$m_1(t) = E(X - t|X > t) \quad = \quad \frac{e^{\lambda(\frac{t}{\alpha})^{\theta}}}{1+\lambda(\frac{t}{\alpha})^{\theta}}\left[\frac{\alpha}{\lambda^{\frac{1}{\theta}}}\Gamma(2+\frac{1}{\theta},\frac{\lambda t^{\theta}}{\alpha^{\theta}}) - t\Gamma(2,\frac{\lambda t^{\theta}}{\alpha^{\theta}})\right]. \tag{3.21}$$

Reversed Hazard rate:

$$\mu(x) = \frac{f(x)}{F(x)} = \frac{\dfrac{\lambda^2\theta x^{2\theta-1}}{(1+\lambda)a^{2\theta}} e^{-\lambda\left\{\left(\frac{x}{a}\right)^{\theta}-1\right\}}}{1-\dfrac{1+\lambda\left(\frac{x}{a}\right)^{\theta}}{1+\lambda} e^{-\lambda\left(\left(\frac{x}{a}\right)^{\theta}-1\right)}} \tag{3.22}$$

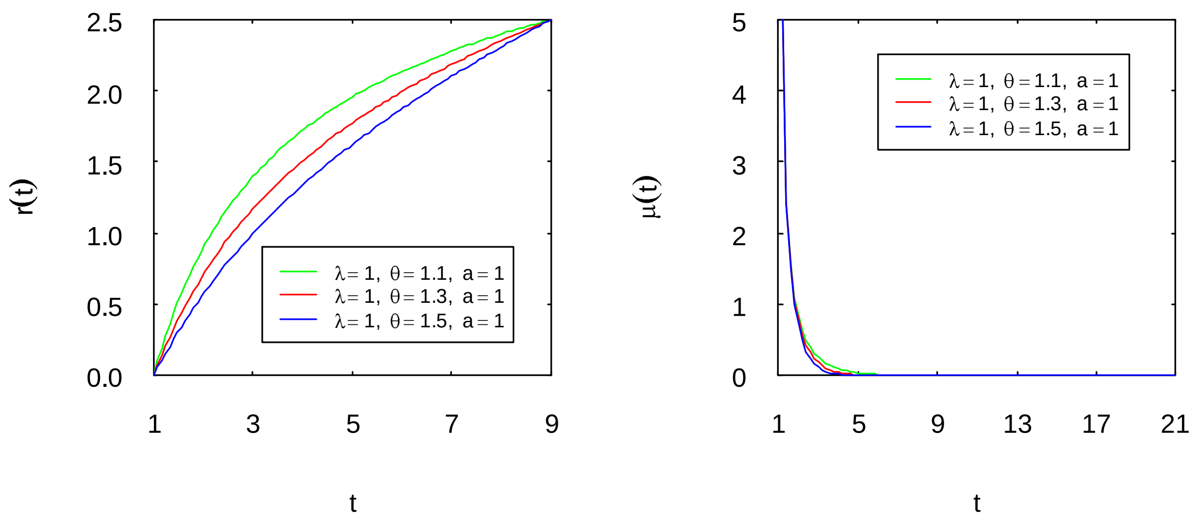

Figure 8: Hazard rate and Reversed Hazard Rate of the OGLPD with θ > 1

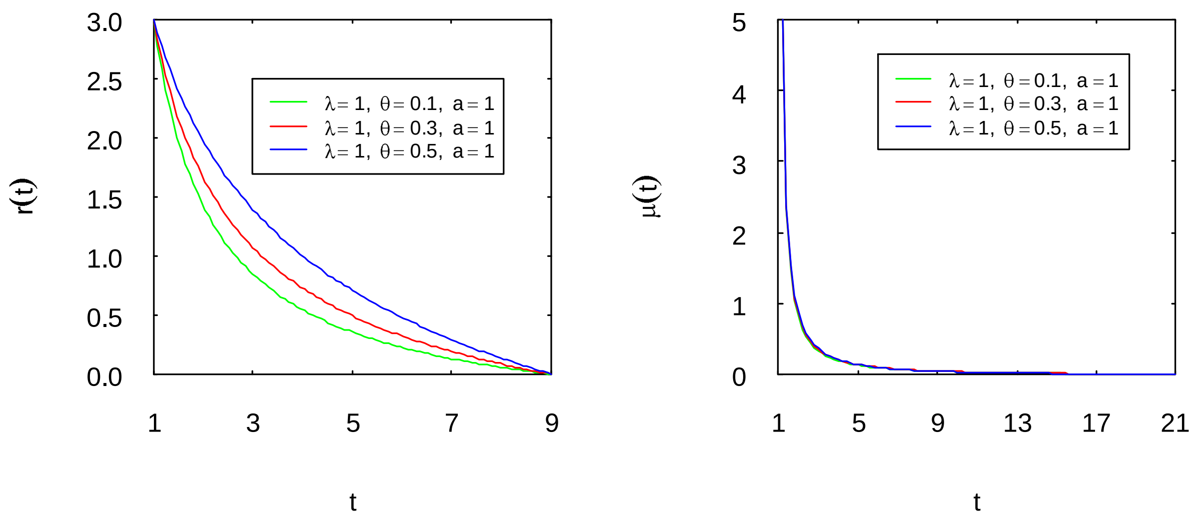

Figure 9: Hazard rate and Reversed Hazard Rate of the OGLPD with θ < 1

Mean Inactivity Time (MIT) or Mean Reversed Residual Life (MRRL) function is defined as

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$$\overline{m}_1(t) = E(t-X|X<t) = \left[t\{\Gamma(2,\lambda)-\Gamma(2,\frac{\lambda t^{\theta}}{\alpha^{\theta}})\} - \frac{\alpha}{\lambda^{\frac{1}{\theta}}}\{\Gamma(2+\frac{1}{\theta},\lambda)-\Gamma(2+\frac{1}{\theta},\frac{\lambda t^{\theta}}{\alpha^{\theta}})\}\right] \cdot \frac{e^{\lambda}}{1+\lambda-[1+\lambda(\frac{t}{\alpha})^{\theta}]e^{-\lambda((\frac{t}{\alpha})^{\theta}-1)}} \quad (3.23)$$

3.7 Stress-Strength Reliability

The Stress-Strength model describes the life of a component which has a random strength X that is subjected to a random stress Y. The component fails at the instant that the stress applied to it exceeds the strength, and the component will function satisfactorily whenever X > Y. So, Stress-

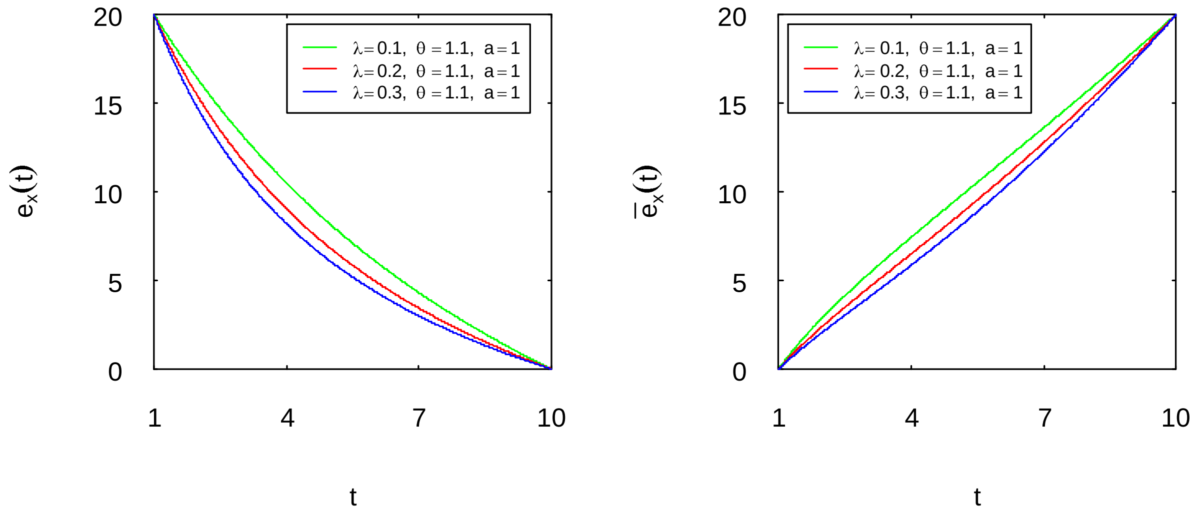

Figure 10: Mean Residual Life (MRL) and Expected Inactivity Time (EIT) of the OGLPD with θ > 1

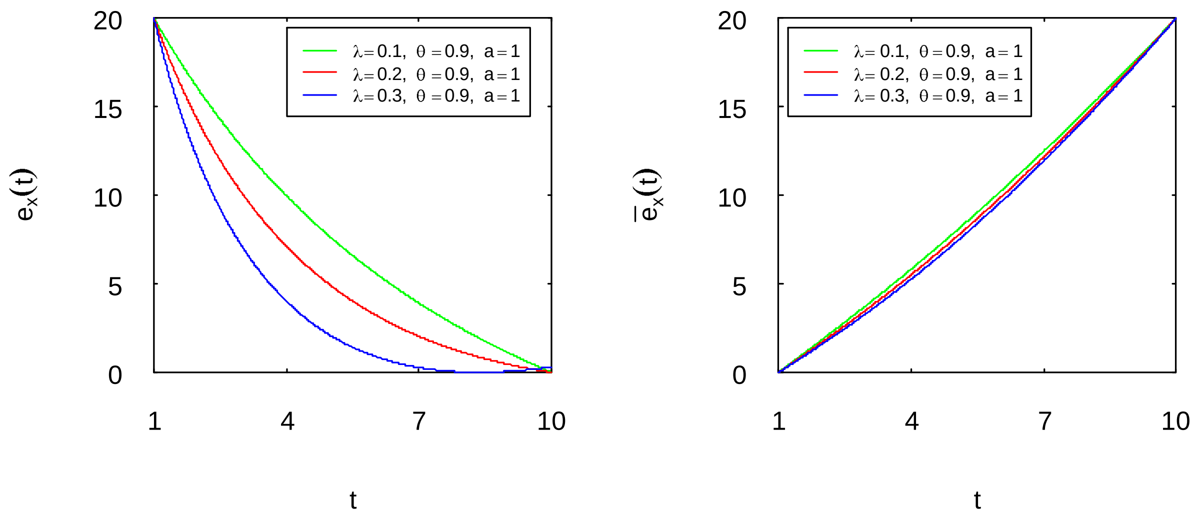

Figure 11: Mean Residual Life (MRL) and Expected Inactivity Time (EIT) of the OGLPD with θ < 1

Strength Reliability is R = P(Y < X). Let, $X_1 \sim OGLPD(\lambda_1,\theta_1,\alpha_1)$ and $X_2 \sim OGLPD(\lambda_2,\theta_2,\alpha_2)$ be independent random variables. Then, the Stress-Strength Reliability is

$$R = P(X_2 < X_1) = 1 - \frac{{\lambda_1}^2\theta_1 e^{\lambda_1+\lambda_2}}{(1+\lambda_1)(1+\lambda_2)\alpha_1^{2\theta_1}} \int_{\alpha_1}^{\infty} [1+\lambda_2(\frac{x}{\alpha_2})^{\theta_2}] x^{2\theta_1-1} e^{-\lambda_1(\frac{x}{\alpha_1})^{\theta_1}-\lambda_2(\frac{x}{\alpha_2})^{\theta_2}} dx$$

If $\theta_1 = \theta_2 = \theta$, then

$$R = 1 - \frac{{\lambda_1}^2 e^{\lambda_1+\lambda_2}}{(1+\lambda_1)(1+\lambda_2)\alpha_1^{2\theta}} \left[\frac{\Gamma(2,\lambda_1+\lambda_2\frac{{\alpha_1}^{\theta}}{{\alpha_2}^{\theta}})}{(\frac{\lambda_1}{\alpha_1^{\theta}}+\frac{\lambda_2}{\alpha_2^{\theta}})^2} + \frac{\lambda_2}{\alpha_2^{\theta}} \frac{\Gamma(3,\lambda_1+\lambda_2\frac{{\alpha_1}^{\theta}}{{\alpha_2}^{\theta}})}{(\frac{\lambda_1}{\alpha_1^{\theta}}+\frac{\lambda_2}{\alpha_2^{\theta}})^3} \right]$$

Also if $\alpha_1 = \alpha_2$, then

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$$R \quad = \quad 1-\frac{{\lambda_1}^2 e^{\lambda_1+\lambda_2}}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2)^2}\left[\Gamma(2,\lambda_1+\lambda_2)+\frac{\lambda_2}{\lambda_1+\lambda_2}\Gamma(3,\lambda_1+\lambda_2)\right]$$

4. Estimation of the Parameters

Here, I estimate the parameters of my model by using the method of Maximum Likelihood Estimation (MLE). The log-likelihood function is

$$\ln L(x;\lambda,\theta,a)=2n\ln\lambda+n\ln\theta+n\lambda-2n\theta\ln a-n\ln(1+\lambda)+(2\theta-1)\sum_{i=1}^{n}\ln x_i-\frac{\lambda}{a^{\theta}}\sum_{i=1}^{n}{x_i}^{\theta}$$

Now the Likelihood function will be maximum at $\hat{a}=x_{(1)}$; the smallest order statistics in the given sample of size n. The MLEs of θ and λ are the roots of the two normal equations.

$$\frac{\delta\ln L(x;\lambda,\theta,\hat{a})}{\delta\theta}=\frac{n}{\theta}-2n\ln a+2\sum_{i=1}^{n}\ln x_i-\lambda\sum_{i=1}^{n}\left(\frac{x_i}{a}\right)^{\theta}\ln\left(\frac{x_i}{a}\right)=0 \qquad (4.24)$$

and

$$\frac{\delta\ln L(x;\lambda,\theta,\hat{a})}{\delta\lambda}=\frac{2n}{\lambda}+n-\frac{n}{1+\lambda}-\sum_{i=1}^{n}\left(\frac{x_i}{a}\right)^{\theta}=0 \qquad (4.25)$$

I estimate of the parameters θ and λ by solving the two equations using numerical method where $\hat{a}=x_{(1)}$.

5. Data Analysis

In this section, I fit the odds generalized Lindley-Pareto model to a real life data set. I consider the data presented by Murthy et al. (2004) on the failure times (in weeks) of 50 components. The data are: 0.013, 0.065, 0.111, 0.111, 0.163, 0.309, 0.426, 0.535, 0.684, 0.747, 0.997, 1.284, 1.304, 1.647, 1.829, 2.336, 2.838, 3.269, 3.977, 3.981, 4.520, 4.789, 4.849, 5.202, 5.291, 5.349, 5.911, 6.018, 6.427, 6.456, 6.572, 7.023, 7.087, 7.291, 7.787, 8.596, 9.388, 10.261, 10.713, 11.658, 13.006, 13.388, 13.842, 17.152, 17.283, 19.418, 23.471, 24.777, 32.795, 48.105. Histogram shows that the data set is positively skewed. Thiago A. N. de Andrade, Marcelo Bourguignon, Gauss M. Cordeiro (2016) fitted this data to the exponentiated generalized extended exponential distribution (EGEE). I have fitted this data set with the Odds Generalized Lindley-Pareto distribution. The estimated values of the parameters were $\lambda = 0.0682$, $\theta = 0.5499$, $\alpha = 0.013$, log-likelihood $=-150.196$ and AIC = 306.391. Histogram and fitted Odds Lindley-Pareto curve to data have been shown in Figure 12.

Table 1: Summarized results of fitting different distributions for the above data set

Distribution	Estimate of the parameters	Likelihood	AIC
EGEED	$\hat{a}=0.3659, \hat{b}=0.3103, \hat{\alpha}=0.3239, \hat{\beta}=0.6041$	-151.150	308.300
OGLPD	$\hat{\lambda}=0.0682, \hat{\theta}=0.5499, \hat{\alpha}=0.013$	-150.196	306.391

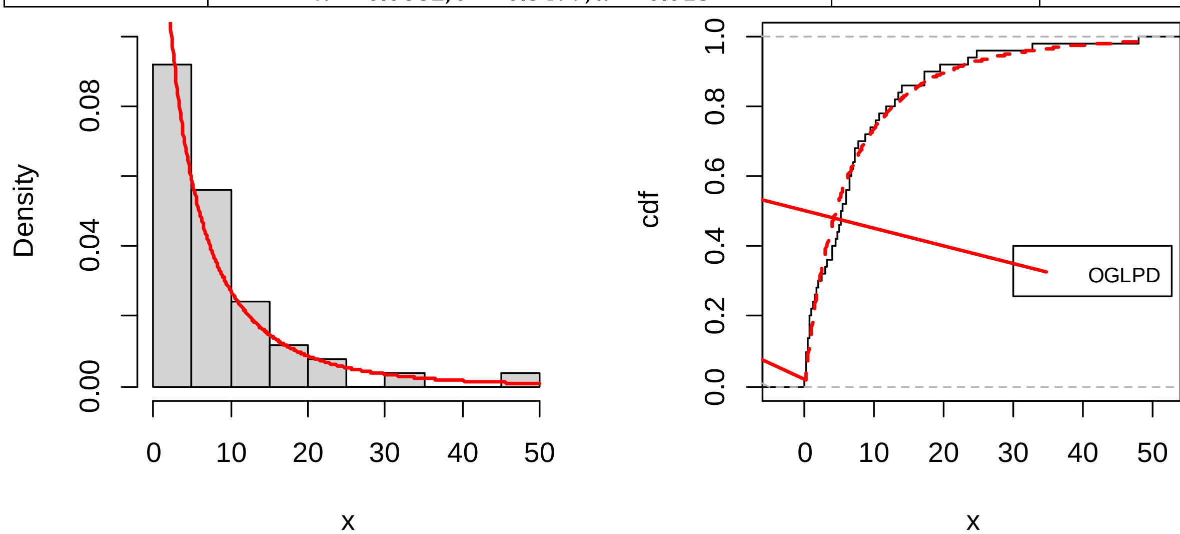

Figure 12: Plots of the estimated pdf and cdf of the OGLPD model for the failure times of 50 components

6. Concluding Remark

In this article, I have studied a new three parameter probability distribution called Odds Generalized Lindley-Pareto Distribution. This is a particular case of Transformed-Transformer (T-X) family of distributions proposed by Alzaatreh et al. (2013). The structural and reliability properties of this distribution have been studied and inference on parameters has also been mentioned. The appropriateness of fitting the Odds Generalized Lindley-Pareto distribution has been established by analyzing a real life data set.

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